The Economics of Monotonicity Conditions: Exploring Choice Incentives in IV Models

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Abstract

This paper examines how to use economic incentives to aid the identification of treatment effects in multi-valued choice models with categorical instrumental variables. We employ a general yet simple framework that utilizes revealed preference analysis to translate choice incentives into identification conditions. We demonstrate that popular identification strategies that rely on monotonicity conditions of the choice equation can be attributed to distinct properties of choice incentives. Additionally, we show that novel identification assumptions emerge when individuals face non-standard choice incentives. We revisit key economic studies in the literature of policy evaluation through the lens of our approach. We then use our framework to study the impact of education on the migration decisions of low-income Mexican households. We use data from Oportunidades, Mexico's largest anti-poverty initiative. We employ incentive analysis to assess the question of whether schooling has a non-monotonic effect on the decision to migrate to the US. We use novel machine learning techniques to estimate causal effects, yielding compelling evidence that completing middle school has a positive effect on migration while additional schooling does not. These findings contribute to a substantial literature that investigate the interplay between education attainment and migration decisions within disadvantaged Mexican households.

Keywords: Revealed Preferences, Causal Inference, Identification, Instrumental Variables, Policy Evaluation.

JEL codes: H43, I18, I38. J38.

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1 Introduction

Economists have long employed instrumental variables (IV) to identify the causal effect of an endogenous treatment variable on an outcome of interest. For an IV to be valid, it must be an exogenous variable that only influences the outcome by affecting the treatment choice. Nevertheless, this property alone is insufficient to ensure the nonparametric identification of treatment effects; additional assumptions are necessary to identify causal parameters. The general approach to secure identification is to impose choice restrictions that limit how agents select the treatments as the instrument varies.

An iconic example of identifying assumption in the binary choice model is the monotonicity condition of Imbens and Angrist (1994). The condition asserts that a change in the instrument induces all agents to alter their choice towards the same direction. Vytlacil (2002) has shown that the monotonicity condition is equivalent to the separability condition which assumes that the treatment choice can be expressed as a latent threshold indicator that compares the propensity score with an unobserved variable that is the source of selection bias (Heckman and Vytlacil, 1999, 2005). These influential ideas spiked a vast literature on both empirical and theoretical aspects of identification assumptions in IV models with binary choices.[1]

The IV literature has successfully extended the notions of monotonicity and separability conditions to the case of multiple-choice models. The seminal work of Angrist and Imbens (1995) extends the previous monotonicity condition of Imbens and Angrist (1994) from the binary choice to the case of the multiple-choice model. Their condition applies to cases where the treatment variable has a natural order. Ordered choice models are also examined by Cameron and Heckman (1998) and further studied by Carneiro et al. (2003); Cunha et al. (2007).

A significant advance in the IV literature is due to Lee and Salanié (2018), who develop general identification results for multiple choice models characterized by a coherent set of separability conditions. On the realm of monotonicity conditions, Heckman and Pinto (2018) propose the unordered monotonicity condition that applies to treatment choices that are not ordered.[2] More recently, Rose and Shem-Tov (2021) proposes a monotonicity condition called extensive margin compliers only (EMCO), in which a change in the instrument incentives all agents to shift from no treatment to some treatment status. Mogstad et al. (2021a,b) investigate the monotonicity criteria in a choice model with multiple instrumental variables.

We propose a departure from the conventional mindset that guides the examination of identification assumptions in IV models. Instead of concentrating on formulating novel monotonicity or

[1]For examples of works in this literature, see Aliprantis (2012); Angrist et al. (2000); Barua and Lang (2016); Dahl et al. (2017); de Chaisemartin (2017); Heckman (2010); Heckman and Urzúa (2010); Heckman and Vytlacil (2007a,b); Huber et al. (2017); Huber and Mellace (2012, 2015); Hull (2018); Imbens and Rubin (1997); Klein (2010); Mogstad et al. (2018); Mogstad and Torgovitsky (2018); Small and Tan (2007).

[2]Unordered choice models have been studied mainly through the literature on structural equations. A common approach assumes that additively separable threshold-crossing models generate the equations that govern the treatment. Examples of this literature are Heckman et al. (2006, 2008); Heckman and Vytlacil (2007a,b).

separability conditions, we explore how economic incentives and classical economic behavior can be used to identify causal parameters in IV models featuring multiple choices and categorical instruments. Our method is rooted in a simple yet general framework that employs revealed preference analysis to transform choice incentives into identification conditions. We demonstrate that distinct patterns of choice incentives can offer economic justification for the identification assumptions frequently invoked in the IV literature. Furthermore, we demonstrate that the framework is a valuable tool for generating novel identification strategies that are economically justified.

The framework offers a distinct advantage: identification relies not on statistical or functional form assumptions, but instead, identification conditions emerge from the application of fundamental principles of economic behavior to choice incentives. This feature enhances the credibility and comprehension of the identification mechanism. The method is flexible enough to accommodate a wide range of non-trivial identification assumptions. We demonstrate its flexibility by examining well-known examples of choice incentives in the literature on policy evaluation. The framework is also capable to provide innovative solutions to non-standard economic scenarios where the identification assumptions typically invoked by the IV literature do not apply.

We use our framework to investigate the migration of poor Mexican households to the US. Currently, the US houses approximately 12 million undocumented residents, with almost half originating from Mexico. Seminal work of Borjas (1987, 1994) suggests a negative selection in migration patterns, with lower-skilled workers benefiting the most from moving to the US. This perspective is supported by Angelucci (2015); Lange (2011), who show that schooling incentives led lower-skilled undocumented migrants to emigrate to the US. She uses data from Oportunidades, Mexico's flagship anti-poverty program (Gertler, 2004). Behrman et al. (2005); Chiquiar and Hanson (2005); Hanson (2006) posits a non-monotonic relationship between education and migration. Fundamental skills such as basic English proficiency taught in middle school increase the propensity to migrate. However, additional education reduces migration, since it makes the domestic labor market more attractive than its international counterpart.

We devise a stylized model that uses data from Oportunidades to assess the question of whether schooling has a non-monotonic effect on the decision to migrate to the US. We use our incentive framework to identify the causal effects of schooling on migration. We find compelling evidence that completing middle school increases the likelihood of migration, while advancing schooling beyond middle school does not. We estimate our model using novel machine learning techniques developed by Navjeevan, Pinto, and Santos (2023).

This paper adds to the economic literature that uses revealed preference analysis to aid the identification of causal parameters in policy evaluations. This recent literature has seen the emergence of seminal works such as Kline and Tartari (2016); Kline and Walters (2016b). In recent years, there has also been a growing interest in applying revealed preference analysis to IV models. This interest is reflected in the increasing number of studies that have employed revealed preference analysis to investigate IV models. Examples of works in this literature include Pinto (2022), Feller et al. (2016), Kamat (2021), Mountjoy (2021), and Brinch et al. (2017). Our empirical analysis adds to a significant literature that evaluates the Oportunidades intervention. Our findings corroborate the hypothesis of a non-monotonic relationship between the migration of poor Mexicans to the US and their education levels. Finally, this paper adds to a growing number of works that apply novel machine learning techniques (Chernozhukov et al., 2022; Smucler et al., 2019) to evaluate data.

This paper is organized as following. Section 2 presents the revealed preference framework and shows how it relates to several works in the literature. Section 3 investigates how patterns of choice incentives yield identification conditions in IV models. Section 4 presents our empirical application. Section 5 concludes.

2 Setup and Notation

Our IV model consists of three observed variables: a categorical instrument Z that takes N_Z values in $\mathcal{Z} = \{z_1, ..., z_{N_Z}\}$; a multiple treatment choice T that takes N_T values in $\mathcal{T} = \{t_1, ..., t_{N_T}\}$; and a real-valued outcome $Y \in \mathbb{R}$. Let $Y(z,t)$ be the counterfactual outcome Y when (Z,T) are fixed to $(z,t) \in \mathcal{Z} \times \mathcal{T}$, and $Y(t)$ be the counterfactual outcome when T is fixed to $t \in \mathcal{T}$.[3] Let $D_t = \mathbf{1}[T=t]; t \in \mathcal{T}$ and $D_z = \mathbf{1}[Z=z]; z \in \mathcal{Z}$ denote binary indicators for treatment choices and IV-values respectively. The *core properties* of IV model for all $(z,t) \in \mathcal{Z} \times \mathcal{T}$ are:

$$\textbf{Exclusion Restriction :} \quad Y(z,t) = Y(t). \tag{1}$$

$$\textbf{IV Exogeneity:} \quad Z \perp\!\!\!\perp (Y(t), T(z)). \tag{2}$$

$$\textbf{IV Relevance:} \quad E\left([D_{z_1}, ..., D_{z_{N_Z}}]'[D_{t_l}, ..., D_{t_{N_T}}]\right) \text{ has full rank.} \tag{3}$$

The exclusion restriction implies that Z affects Y only through T. The exogeneity assumption means that the instrument Z is as good as randomly assigned, and the IV relevance states that Z causes T. All variables belong to the probability space $(\mathcal{I}, \mathcal{F}, P)$ where $i \in \mathcal{I}$ denotes an individual. We suppress baseline variables $\boldsymbol{X}$ for notational simplicity. All analyses can be understood as conditioned on $\boldsymbol{X}$.

The *response vector* $\boldsymbol{S} \equiv [T(z_1), ..., T(z_{N_Z})]'$ is the unobserved N_Z-dimensional vector that stacks the counterfactual choices $T(z)$ across the IV-values z in $\mathcal{Z}$. Elements $\boldsymbol{s}$ in the support of the response vector, $\mathcal{S} = \{\boldsymbol{s}_1, ..., \boldsymbol{s}_{N_S}\}$, are called *response types* or simply *types*. To fix ideas, consider the Local Average Treatment Effects (LATE) model of Imbens and Angrist (1994) with a binary instrument $Z \in \{z_0, z_1\}$ and a binary treatment $T \in \{t_0, t_1\}$. The response vector $\boldsymbol{S} = [T(z_0), T(z_1)]'$ admits four possible types: never-takers $\boldsymbol{s}_{\text{nt}} = [t_0, t_0]'$, compliers $\boldsymbol{s}_{\text{c}} = [t_0, t_1]'$, always-takers $\boldsymbol{s}_{\text{at}} = [t_1, t_1]'$, and defiers $\boldsymbol{s}_{\text{d}} = [t_1, t_0]'$.

The response vector enables us to connect observed quantities with the unobserved counter-

[3] This notation uses the potential outcome framework of Holland (1986); Rubin (1978). For a discussion on causality and the fixing operation, see Heckman and Pinto (2013, 2022).

factuals according to the following equation:[4]

$$\underbrace{E(Y|T=t,Z=z)P(T=t|Z=z)}_{\text{Observed}}=\sum_{\boldsymbol{s}\in\mathcal{S}}\underbrace{\mathbf{1}[T=t|\boldsymbol{S}=\boldsymbol{s},Z=z]}_{\text{Known}}\cdot\underbrace{E(Y(t)|\boldsymbol{S}=\boldsymbol{s})P(\boldsymbol{S}=\boldsymbol{s})}_{\text{Unobserved}}\ \forall(z,t)\in\mathcal{Z}\times\mathcal{T}. \tag{4}$$

The left-hand side of equation (4) comprises of the observed quantities, namely, the conditional expectation $E(Y|T=t,Z=z)$ and propensity score $P(T=t|Z=z)$.[5] The first term of the right-hand side of the equation is nonrandom since choice T is a fully determined given IV-value z and type $\boldsymbol{s}$. The second term on the right-hand side is unobserved. It comprises the expected value of counterfactual outcomes conditioned on response types $E(Y(t)|\boldsymbol{S}=\boldsymbol{s})$ and type probabilities $P(\boldsymbol{S}=\boldsymbol{s})$.

Equation (4) poses a fundamental identification problem. It establishes a system of linear equations in which the right-hand side comprises $N_Z\cdot N_T$ observed quantities while the left-hand side consists of unobserved quantities that depend on the number of response types N_S. Without additional assumptions, the total number of types amounts to $N_T^{N_Z}$, which precludes the point-identification of the unobserved quantities. Consequently, the identification of causal parameters hinges on assumptions that limit the number of admissible types.

A well-known identification assumption in the LATE model is the monotonicity condition (Imbens and Angrist, 1994). The condition translates choice incentives into a choice inequality. Namely, if IV-value z_1 incentivizes the treatment status t_1, then it is natural to assume that a change in the instrument from z_0 to z_1 induces the agents towards choosing t_1. The condition is formalized as:

$$\mathbf{1}[T_i(z_0)=t_1]\leq\mathbf{1}[T_i(z_1)=t_1]\ \forall i. \tag{5}$$

The monotonicity condition eliminates the defiers ($\boldsymbol{s}_d$), which secures the identification of the causal effect for the compliers $E(Y(t_h)-Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_c)$. We characterize the types of a choice model via a response matrix $\boldsymbol{R}$ whose columns denote types and each row corresponds to an IV-value. The response matrix of the LATE model is:

$$\boldsymbol{R}=\begin{array}{cc} & \begin{array}{ccc}\boldsymbol{s}_{nt} & \boldsymbol{s}_c & \boldsymbol{s}_{at}\end{array} \\ & \left[\begin{array}{ccc} t_0 & t_0 & t_1 \\ t_0 & t_1 & t_1\end{array}\right]\begin{array}{c}T(z_0)\\T(z_1)\end{array}\end{array} \tag{6}$$

[4]This equation is proved in Heckman and Pinto (2018).

[5]Equation (4) holds for any real-valued function $g:\mathbb{R}\to\mathbb{R}$ and for $(z,t)\in\mathcal{Z}\times\mathcal{T}$, that is:

$$E(g(Y)|T=t,Z=z)P(T=t|Z=z)=\sum_{\boldsymbol{s}\in\mathcal{S}}\mathbf{1}[T=t|\boldsymbol{S}=\boldsymbol{s},Z=z]\cdot E(g(Y(t))|\boldsymbol{S}=\boldsymbol{s})P(\boldsymbol{S}=\boldsymbol{s}).$$

Setting $g(Y)=\mathbf{1}[Y=y];y\in\mathbb{R}$ generates an equation for the probabilities of counterfactual outcomes. Setting $g(Y)=1$ generates an equation that relates propensity scores and response type probabilities.

2.1 Exploring Choice Incentives

Extending the monotonicity condition of Imbens and Angrist (1994) to multi-treatment scenarios presents significant challenges. While the binary treatment case yields a singular, well-defined monotonicity condition, the multiple-choice model introduces a spectrum of possible conditions (Lee and Salanié, 2018). Notably, the monotonicity proposed by Angrist and Imbens (1995) and the unordered monotonicity outlined by Heckman and Pinto (2018) differ markedly in multi-choice settings. However, both conditions collapse to the same monotonicity criteria specified in (5) when the treatment is binary. We make the case that the study of choice incentives provides a powerful framework to investigate, generate, and justify monotonicity conditions in multiple choice models. Some notation is needed to translate choice incentives into monotonicity conditions.

The *incentive matrix* $\boldsymbol{L}$ is a $N_Z \times N_T$-dimensional matrix that characterizes the choice incentives induced by the instrument. Each input $\boldsymbol{L}[z,t]$ denotes the relative incentive of the IV-value z (row) towards choice $t \in \mathcal{T}$ (column). The inequality $\boldsymbol{L}[z,t] \leq \boldsymbol{L}[z',t]$ means that a change in the IV from z to z' increases the incentives towards choice t, making the choice more attractive. Appendix A.1 uses revealed preference analysis to justify the following choice rule:

$$\textbf{Choice Rule:} \quad \text{If } T_i(z) = t \text{ and } \boldsymbol{L}[z',t'] - \boldsymbol{L}[z,t'] \leq \boldsymbol{L}[z',t] - \boldsymbol{L}[z,t] \text{ then } T_i(z') \neq t'. \tag{7}$$

The Choice Rule states that if agent i prefers choice t over t' under z-incentives ($T_i(z) = t$), and if z'-incentives favor t at least as much as t' ($\boldsymbol{L}[z',t'] - \boldsymbol{L}[z,t'] \leq \boldsymbol{L}[z',t] - \boldsymbol{L}[z,t]$), then the agent will not choose t' over t ($T_i(z') \neq t'$). This rule highlights a cornerstone principle of rational choice theory: an individual's preferences will remain consistent unless there is a compelling incentives to choose otherwise. In simple terms, it means that the individual will change its choice only it has incentives to do so. The choice rule is useful to translates choice incentives into choice restrictions that, in turn, eliminate response types. We illustrate the application of the choice rule by revisiting primary examples from the IV literature.

Example **E.1**. Our methodology offers an economic justification for the monotonicity condition in the LATE model. The condition emerges when applying revealed preference analysis, neatly captured by the choice rule, to examine choice incentives represented by its incentive matrix. Specifically, let the IV value z_1 incentivizes choice t_1, while z_0 plays the role of a baseline comparison that does not incentivize any of the choices t_0, t_1. The corresponding incentive matrix is given by:

$$\boldsymbol{L} = \begin{array}{c c} \begin{array}{cc} t_0 & t_1 \end{array} & \\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} & \begin{array}{c} z_0 \\ z_1 \end{array} \end{array} \tag{8}$$

The first row of the matrix takes values zero since z_0 offers no incentives while the second row indicates that z_1 incentivizes t_1. We use the choice rule to compare t_1 versus t_0 when the IV changes from z_0 to z_1 :

$$T_i(z_0) = t_1, \text{ and } \boldsymbol{L}[z_1, t_0] - \boldsymbol{L}[z_0, t_0] = 0 \leq 1 = \boldsymbol{L}[z_1, t_1] - \boldsymbol{L}[z_0, t_1] \text{ thus } T_i(z_1) \neq t_0.$$

In summary, the LATE incentives lead to the choice restriction $T_i(z_0) = t_1 \Rightarrow T_i(z_1) \neq t_0$. It states that if agent i chooses t_1 under no incentives (z_0), it will not choose t_0 when the incentives for t_1 are present (z_1). This restriction eliminates the defiers and is equivalent to assume the customary monotonicity condition in (5). □

Next example investigates a multiple choice model.

Example **E.2**. Kline and Walters (2016a) evaluate the Head Start Impact Study using a model with three treatment choices $T \in \{n, c, h\}$ corresponding to three day-care options: h stands for Head Start, c for other preschool programs, and n for no preschool (home-care). There are two instrumental values $Z \in \{z_0, z_1\}$, with z_1 indicating an offer to attend a Head Start school and z_0 if no offer is granted. The authors assume that the offer to attend Head Start offer does not induce children to leave Head Start nor instigate the children to switch between c and n. They express these assumptions by the following choice restriction:

$$T_i(z_0) \neq T_i(z_1) \Rightarrow T_i(z_1) = h \, \forall \, i \in \mathcal{I}. \tag{9}$$

This restriction can also be obtained by applying the choice rule to the model incentives. Specifically, the incentive matrix of the choice model is:

$$\boldsymbol{L} = \begin{array}{c} \begin{matrix} n & c & h \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ z_0 \\ z_1 \end{array} \tag{10}$$

We can apply the choice rule in four instances:

$$\begin{aligned} T_i(z_0) = h, &\text{ and } \boldsymbol{L}[z_1, c] - \boldsymbol{L}[z_0, c] = 0 \leq 1 = \boldsymbol{L}[z_1, h] - \boldsymbol{L}[z_0, h] \text{ thus } T_i(z_1) \neq c, \\ T_i(z_0) = h, &\text{ and } \boldsymbol{L}[z_1, n] - \boldsymbol{L}[z_0, n] = 0 \leq 1 = \boldsymbol{L}[z_1, h] - \boldsymbol{L}[z_0, h] \text{ thus } T_i(z_1) \neq n, \\ T_i(z_0) = n, &\text{ and } \boldsymbol{L}[z_1, c] - \boldsymbol{L}[z_0, c] = 0 \leq 0 = \boldsymbol{L}[z_1, n] - \boldsymbol{L}[z_0, n] \text{ thus } T_i(z_1) \neq c, \\ T_i(z_0) = c, &\text{ and } \boldsymbol{L}[z_1, n] - \boldsymbol{L}[z_0, n] = 0 \leq 0 = \boldsymbol{L}[z_1, c] - \boldsymbol{L}[z_0, c] \text{ thus } T_i(z_1) \neq n. \end{aligned}$$

The first and second choice restrictions are summarized by $T_i(z_0) = h \Rightarrow T_i(z_1) = h$. The third restriction is $T_i(z_0) = n \Rightarrow T_i(z_1) \neq c$, and the fourth is $T_i(z_0) = c \Rightarrow T_i(z_1) \neq n$. Altogether, these restrictions are equivalent to the author's restriction in (9). These restrictions eliminate four of the nine possible types. The five response types that survive the elimination process are displayed in the following response matrix:

$$\boldsymbol{R} = \begin{array}{c} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 \end{matrix} \\ \begin{bmatrix} n & c & h & n & c \\ n & c & h & h & h \end{bmatrix} \end{array} \begin{array}{l} \\ T(z_0) \\ T(z_1) \end{array} \quad \square$$

A potential critique to the revealed preference methodology described here is that it demands additional machinery that may be excessive to investigate simple models such as LATE. In such cases, it is easier to simply invoking the monotonicity condition. The benefits of the revealed preference framework become more salient when examining more complex models. It is often the case that the revealed preference approach outperforms the analyses based solely on monotonicity conditions. The next examples illustrate such cases.

Example **E.3**. Kirkeboen, Leuven, and Mogstad (2016) investigate a choice model featuring three treatment options (t_0, t_1, t_2) and three IV-values (z_0, z_1, z_2). In this model, z_1 incentivizes choice t_1, z_2 incentivizes t_2, and z_0 provides no incentives. The response vector is denoted by $\boldsymbol{S} = [T(z_0), T(z_1), T(z_2)]'$. There are a total of 27 potential response types since each of the three counterfactual choices $(T(z_0), T(z_1), T(z_2))$ can take on any of the three treatment values (t_0, t_1, t_2). The choice incentives justify two monotonicity conditions:

$$\mathbf{1}[T_i(z_0) = t_1] \leq \mathbf{1}[T_i(z_1) = t_1], \quad \text{and} \quad \mathbf{1}[T_i(z_0) = t_2] \leq \mathbf{1}[T_i(z_2) = t_2]. \tag{11}$$

The first condition states that an IV-change from z_0 to z_1 induces agents to shift their choice toward t_1 while the second condition states that a change from z_0 to z_2 induces agents toward t_2. These monotonicity conditions eliminate 12 response types,[6] which is not sufficient to ensure the point-identification of any counterfactual outcome. The revealed preference approach delivers more identification power. The corresponding incentive matrix and the choice restrictions generated by the choice rule (7) are displayed below:

$$\boldsymbol{L} = \begin{array}{c c} \begin{array}{ccc} t_0 & t_1 & t_2 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} z_0 \\ z_1 \\ z_2 \end{array} \end{array} \;\therefore\; \begin{array}{lcl} T_i(z_0) = t_0 & \Rightarrow & T_i(z_1) \neq t_2 \text{ and } T_i(z_2) \neq t_1 \\ T_i(z_0) = t_1 & \Rightarrow & T_i(z_1) = t_1 \text{ and } T_i(z_2) \neq t_0 \\ T_i(z_0) = t_2 & \Rightarrow & T_i(z_1) \neq t_0 \text{ and } T_i(z_2) = t_2 \\ T_i(z_1) = t_2 & \Rightarrow & T_i(z_0) = t_2 \text{ and } T_i(z_2) = t_2 \\ T_i(z_2) = t_1 & \Rightarrow & T_i(z_0) = t_1 \text{ and } T_i(z_1) = t_1 \end{array} \tag{12}$$

The incentive matrix $\boldsymbol{L}$ denotes that z_1 incentivizes t_1 while the last row means that z_2 incentivizes t_2. Appendix B applies choice rule (7) to each combination of two treatment values $(t, t') \in \{t_0, t_1, t_2\}$ and two instrumental values $(z, z') \in \{z_0, z_1, z_2\}^2$, which results in the five choice restrictions displayed above. The restrictions are intuitive. The first choice restriction states that if an agent chooses t_0 under z_0 (no incentives), then it will not choose t_2 under z_1, since z_1 does not incentivize t_2. The agent will not choose t_1 under z_2 either since z_2 does not incentivize t_1 either. In total, the five choice restrictions eliminate 19 response types, including the 12 types eliminated by the monotonicity conditions in (11).[7] The eight types that survive the elimination process are displayed in the following response matrix:

$$\boldsymbol{R} = \begin{array}{c c} \begin{array}{cccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 \end{array} & \\ \begin{bmatrix} t_0 & t_1 & t_2 & t_0 & t_0 & t_0 & t_1 & t_2 \\ t_0 & t_1 & t_2 & t_1 & t_0 & t_1 & t_1 & t_1 \\ t_0 & t_1 & t_2 & t_2 & t_2 & t_0 & t_2 & t_2 \end{bmatrix} & \begin{array}{c} T(z_0) \\ T(z_1) \\ T(z_2) \end{array} \end{array} \tag{13}$$

[6] The first monotonicity condition eliminates the six types given by $[t_1, t_2, t']$ or $[t_1, t_3, t']$ for $t' \in \{t_0, t_1, t_2\}$. The second monotonicity condition eliminates another six types: $[t_2, t', t_1]$ or $[t_2, t', t_3]$ for $t' \in \{t_0, t_1, t_2\}$. See Appendix B for this analysis.

[7] The elimination process is presented in Table (A.1) of Appendix B.

The first three response types, $\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{s}_3$, correspond to always-takers. They refer to agents that choose the same treatment choice (t_0, t_1, t_2 respectively) regardless of the instrumental value. Type $\boldsymbol{s}_4$ is called a full complier. It refers to agents that are most responsive to the IV incentives. They choose t_0 under no incentives, t_1 when assigned to z_1, and t_2 when assigned to z_2. The remaining four types $\boldsymbol{s}_5, ..., \boldsymbol{s}_8$ are called partial compliers since they choose two out of the three possible treatment statuses. □

Example **E.4**. Pinto (2022) examines the housing experiment called Moving to Opportunity. The model consists of three neighborhood choice: t_h, t_m, and t_l denote high-, medium-, and low-poverty neighborhoods respectivelly. Families were randomly assigned to one of the three groups: the control group z_c offers no incentives, the Section Eight group z_8 received a housing voucher that incentivized families to choose either medium-poverty (t_m) or low-poverty (t_l) neighborhoods; the Experimental group z_e received a voucher that incentivized families to live in a low-poverty (t_l) neighborhoods; and the control group z_c received no voucher. These incentives justify three monotonicity conditions:

$$\mathbf{1}[T_i(z_c) = t_l] \leq \mathbf{1}[T_i(z_e) = t_l], \quad \mathbf{1}[T_i(z_c) \in \{t_m, t_l\}] \leq \mathbf{1}[T_i(z_8) \in \{t_m, t_l\}], \text{ and } \mathbf{1}[T_i(z_e) = t_m] \leq \mathbf{1}[T_i(z_8) = t_m].$$

The first condition means that an IV change from z_c to z_e promotes t_h since z_c offers no incentives and z_e incentivizes only t_l. The second means that a change from z_c to z_8 incite choices t_m or t_h, since z_8 incentivizes both t_m and $_l$. The last condition means that a change from z_e to z_8 instigate choice t_m since both z_e, z_8 incentivize t_l but only z_8 incentivizes t_m. These conditions eliminate 14 out of the 27 response types which do not secure the point-identification of response type probabilities or counterfactual outcomes. The revealed preference analysis is more effective at eliminating response types. The incentive matrix and the corresponding choice restrictions are displayed below:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} t_h & t_m & t_l \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_c \\ z_8 \\ z_e \end{array} \quad \therefore \quad \begin{array}{lcl} T_i(z_c) = t_l & \Rightarrow & T_i(z_e) = t_l \text{ and } T_i(z_8) \neq t_h \\ T_i(z_c) = t_m & \Rightarrow & T_i(z_e) \neq t_h \text{ and } T_i(z_8) \neq t_h \\ T_i(z_e) = t_m & \Rightarrow & T_i(z_c) = t_m \text{ and } T_i(z_8) = t_m \\ T_i(z_e) = t_h & \Rightarrow & T_i(z_c) = t_h \text{ and } T_i(z_8) \neq t_l \\ T_i(z_8) = t_h & \Rightarrow & T_i(z_c) = t_h \text{ and } T_i(z_e) = t_h \\ T_i(z_8) = t_l & \Rightarrow & T_i(z_e) = t_l \\ T_i(z_c) \neq t_h & \Rightarrow & T_i(z_8) = T_i(z_c) \end{array} \tag{14}$$

These restrictions eliminate 20 out of the 27 possible response types including those eliminated by the monotonicity conditions. The seven types that survive the elimination process are displayed in the following response matrix:

$$\boldsymbol{R} = \begin{array}{c} \begin{array}{ccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 \end{array} \\ \begin{bmatrix} t_h & t_m & t_l & t_h & t_h & t_m & t_h \\ t_h & t_m & t_l & t_m & t_l & t_m & t_m \\ t_h & t_m & t_l & t_l & t_l & t_l & t_h \end{bmatrix} \end{array} \begin{array}{c} \\ T(z_c) \\ T(z_8) \\ T(z_e) \end{array} \tag{15}$$

These response types enable the point-identification of all response type probabilities and most of the counterfactual outcomes. □

Example **E.5**. Mountjoy (2022) studies the returns to two- and four-year college degrees and uses the proximity to college as an IV to encourage college enrollment. Let $T \in \{0, 2, 4\}$ represent the number of years of the college degree. The discrete version of the instrument is $Z = (Z_2, Z_4) \in \{0, 1\}^2$, where Z_2 and Z_4 indicate the proximity to two-year and four-year colleges, respectively. We use $T(z_2, z_4)$ for the counterfactual choice, and the response vector $\boldsymbol{S} = [T(0,0), T(0,1), T(1,0), T(1,1)]'$ can take on the values of 81 potential response types. Proximity serves as an incentive for college enrollment, thereby justifying six natural monotonicity conditions:

$$\begin{aligned}
&\mathbf{1}[T_i(1, z_4) = 0] \leq \mathbf{1}[T_i(0, z_4) = 0], \qquad && \mathbf{1}[T_i(z_2, 1) = 0] \leq \mathbf{1}[T_i(z_2, 0) = 0],\\
&\mathbf{1}[T_i(1, z_4) = 2] \geq \mathbf{1}[T_i(0, z_4) = 2], \qquad && \mathbf{1}[T_i(z_2, 1) = 2] \leq \mathbf{1}[T_i(z_2, 0) = 2],\\
&\mathbf{1}[T_i(1, z_4) = 4] \geq \mathbf{1}[T_i(0, z_4) = 4], \qquad && \mathbf{1}[T_i(z_2, 1) = 4] \geq \mathbf{1}[T_i(z_2, 0) = 4].
\end{aligned}$$

These monotonicity conditions state that an increase in the proximity to a two-year college induces agents towards choice 2 and away from choices 0 and 4. Conversely, an increase in the proximity to a four-year college induces agents towards choice 4 and away from choices 0 and 2. These monotonicity conditions eliminate 70 out of the 81 possible response types. The revealed preference analysis is capable of eliminating additional types. The incentive matrix of this choice model and its corresponding choice restrictions generated by the choice rule (7) are as follows:

$$\boldsymbol{L} = \begin{array}{c c} \begin{array}{ccc} 0 & 2 & 4 \end{array} & (z_2, z_4) \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} & \begin{array}{c} (0,0) \\ (0,1) \\ (1,0) \\ (1,1) \end{array} \end{array} \quad \therefore \quad \begin{array}{lcl}
T_i(1,1) = 0 & \Rightarrow & T_i(0,0) = 0, \quad T_i(0,1) = 0, \quad T_i(1,0) = 0 \\
T_i(0,1) = 0 & \Rightarrow & T_i(0,0) = 0 \\
T_i(1,0) = 0 & \Rightarrow & T_i(0,0) = 0 \\
T_i(0,1) = 2 & \Rightarrow & T_i(1,1) = 2, \quad T_i(1,0) = 2, \quad T_i(0,0) = 2 \\
T_i(0,0) = 2 & \Rightarrow & T_i(1,0) = 2, \quad T_i(1,1) = 2 \\
T_i(1,1) = 2 & \Rightarrow & T_i(1,0) = 2 \\
T_i(1,0) = 4 & \Rightarrow & T_i(1,1) = 4, \quad T_i(0,0) = 4, \quad T_i(0,1) = 4 \\
T_i(0,0) = 4 & \Rightarrow & T_i(0,1) = 4, \quad T_i(1,1) = 4 \\
T_i(1,1) = 4 & \Rightarrow & T_i(0,1) = 4
\end{array} \tag{16}$$

The incentive matrix generates nine choice restrictions that eliminate 72 response types, including those eliminated by the monotonicity conditions. The response matrix containing the nine types that survive this elimination process is displayed below:

$$\boldsymbol{R} = \begin{array}{cc} \begin{array}{ccccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 & \boldsymbol{s}_9 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2 & 2 & 4 & 4 \\ 0 & 0 & 4 & 4 & 4 & 2 & 4 & 4 & 4 \\ 0 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 4 \\ 0 & 2 & 4 & 2 & 4 & 2 & 2 & 4 & 4 \end{bmatrix} & \begin{array}{c} T(0,0) \\ T(0,1) \\ T(1,0) \\ T(1,1) \end{array} \end{array}$$

□

2.2 Identification of Causal Parameters

The response matrix is pivotal in the causal analysis of a choice model because it contains all the essential information to analyze the identification of type probabilities and counterfactual outcomes (Heckman and Pinto, 2018). We seek to characterizes which causal parameters are identified for a given response matrix $\boldsymbol{R}$. To do so, it is useful to express equation (4) using the following matrix representation:

$$\boldsymbol{Q}_Z(t) \odot \boldsymbol{P}_Z(t) = \boldsymbol{B}_t \cdot \left(\boldsymbol{Q}_S(t) \odot \boldsymbol{P}_S\right) \quad \text{for all } t \in \mathcal{T}, \tag{17}$$

where $\boldsymbol{Q}_Z(t) \equiv [E(Y|T=t, Z=z_1), ..., E(Y|T=t, Z=z_{N_Z})]'$ is the observed vector of outcome expectations; $\boldsymbol{P}_Z(t) \equiv [P(T=t|Z=z_1), ..., P(T=t|Z=z_{N_Z})]$ is the observed vector of propensity scores; $\boldsymbol{Q}_S(t) \equiv [E(Y(t)|\boldsymbol{S}=\boldsymbol{s}_1), ..., E(Y(t)|\boldsymbol{S}=\boldsymbol{s}_{N_S})]$, is the unobserved vector of counterfactual outcomes; $\boldsymbol{P}_S \equiv [P(\boldsymbol{S}=\boldsymbol{s}_1), ..., P(\boldsymbol{S}=\boldsymbol{s}_{N_S})]$ is the unobserved vector of type probabilities; and $\odot$ denotes element-wise (Hadamard) multiplication. Finally, $\boldsymbol{B}_t \equiv \mathbf{1}[\boldsymbol{R}=t]$ is the $N_Z \times N_S$ binary matrix that takes value one if the entry in $\boldsymbol{R}$ is t and zero otherwise. Matrices $\boldsymbol{B}_t$ usually have full row rank since the number of columns N_S far exceeds the number of rows N_Z. We also use $\boldsymbol{B}_t[\cdot, \boldsymbol{s}]$ and $\boldsymbol{B}_t[z, \cdot]$ for the $\boldsymbol{s}$-column and z-row of $\boldsymbol{B}_t$ respectively. Under this notation, we can state the following identification result:[8]

Theorem T.1. Let $\boldsymbol{R}$ denotes a response matrix for a choice model in which IV Assumptions (1)–(3) hold and let $\tilde{\mathcal{S}} \subset \mathcal{S}$ be a subset of response types. For any choice $t \in \mathcal{T}$, such that the binary matrix $\boldsymbol{B}_t = \mathbf{1}[\boldsymbol{R}=t]$ has full row rank, we have that:

$$E(Y(t)|\boldsymbol{S} \in \tilde{\mathcal{S}}) \text{ is identified} \quad \Leftrightarrow \quad \frac{\left(\sum_{\boldsymbol{s}\in\tilde{\mathcal{S}}} \boldsymbol{B}_t[\cdot,\boldsymbol{s}]\right)'\left(\boldsymbol{B}_t\boldsymbol{B}_t'\right)^{-1}\left(\sum_{\boldsymbol{s}\in\tilde{\mathcal{S}}} \boldsymbol{B}_t[\cdot,\boldsymbol{s}]\right)}{|\tilde{\mathcal{S}}|} = 1,$$

where $|\tilde{\mathcal{S}}|$ is the number of response types in the set $\tilde{\mathcal{S}}$. Moreover, if $E(Y(t)|\boldsymbol{S} \in \tilde{\mathcal{S}})$ is identified, it can be evaluated by:

$$P(\boldsymbol{S} \in \tilde{\mathcal{S}}) = \Big(\sum_{\boldsymbol{s}\in\tilde{\mathcal{S}}} \boldsymbol{B}_t[\cdot,\boldsymbol{s}]\Big)' \left(\boldsymbol{B}_t\boldsymbol{B}_t'\right)^{-1} \boldsymbol{P}_Z(t),$$

$$E(Y(t)|\boldsymbol{S} \in \tilde{\mathcal{S}})P(\boldsymbol{S} \in \tilde{\mathcal{S}}) = \Big(\sum_{\boldsymbol{s}\in\tilde{\mathcal{S}}} \boldsymbol{B}_t[\cdot,\boldsymbol{s}]\Big)' \left(\boldsymbol{B}_t\boldsymbol{B}_t'\right)^{-1} \left(\boldsymbol{Q}_Z(t) \odot \boldsymbol{P}_Z(t)\right).$$

Proof. See Appendix A.3 □

The theorem presents a simple criterion to check if a counterfactual outcome $Y(t)$ is identified for an arbitrary set of types $\tilde{\mathcal{S}}$. Given the assumptions of the theorem, we can say that for any choice $t \in \mathcal{T}$, and any response type $\boldsymbol{s} \in \mathcal{S}$, $E(Y(t)|\boldsymbol{S}=\boldsymbol{s})$ is identified if and only if $\boldsymbol{B}_t[\cdot,\boldsymbol{s}]' \left(\boldsymbol{B}_t\boldsymbol{B}_t'\right)^{-1} \boldsymbol{B}_t[\cdot,\boldsymbol{s}] = 1$.[9]

2.3 Generality and Limitations

The framework presented here broadly applies to IV models that can be characterized by an incentive matrix. This scope encompasses instruments that span a wide range of incentives, which serve to augment the attractiveness, accessibility, or affordability of the treatment choices. Examples of such instruments include, but are not limited to, monetary rewards, advertising campaigns, tax reductions, subsidies, pricing mechanisms, and geographical proximity.

[8] The theorem also holds for the outcome transformation $g(Y)$ for any function $g: \mathbb{R} \to \mathbb{R}$.

[9] Appendix A.4 illustrates this result by applying the theorem to the LATE model.

The incentive matrix exhibits several desirable properties. It allows an IV value to incentivize more than one treatment choice. The choice restrictions generated by an incentive matrix remain the same under its row and column permutations. Choice restrictions are also invariant to any monotonic transformations of the incentive matrix. The instrument is said to offer the same incentive for choices t and t' if the corresponding columns of the incentive matrix are equal. In this case, the choice restrictions and the response matrix generated by the incentive matrix are symmetric with respect to these choices. Identical rows of an incentive matrix mean that the corresponding IV-values are distinguishable in terms of choice incentives. In this case, it is advisable to merge these IV-values into a single representative value.

A key requirement of the incentive matrix is that treatment incentives must be comparable. For instance, the incentive matrix is not suitable to represent the choice incentives of a schooling experiment that seeks to boost academic performance by offering students monetary prizes or academic awards. These incentives are not easily ranked since some students may favor academic awards, while others gravitate towards monetary benefits.

3 Which Incentives Justify Monotonicity Conditions?

Our framework enables us to investigate the economic content of monotonicity conditions typically invoked in the IV literature. We show that monotonicity conditions can be traced back to specific patterns of choice incentives. In particular, we seek to examine the pattern of choice incentives that justifies the well-known monotonicity condition of Angrist and Imbens (1995), the Unordered monotonicity of Heckman and Pinto (2018), and the EMCO condition of (Andresen and Huber, 2021; Angrist and Imbens, 1995; Rose and Shem-Tov, 2021).

3.1 Investigating Ordered Monotonicity

Angrist and Imbens (1995) extend of the monotonicity of Imbens and Angrist (1994) to the case of an ordered multiple treatment. The condition states that a change in the instrument induces all agents towards the same treatment direction:

$$T_i(z) \leq T_i(z') \, \forall \, i, \text{ or } T_i(z) \geq T_i(z') \, \forall \, i \text{ and any } z, z' \in \mathcal{Z}. \tag{18}$$

A celebrated result of Angrist and Imbens (1995) is that their monotonicity condition yields a causal interpretation to the standard Two-Stage Least Squares (2SLS) estimator. Vytlacil (2006) shows that their monotonicity condition is equivalent to assuming an ordered choice model with random thresholds. This monotonicity condition is commonly perceived as an intrinsic attribute of treatment choices that exhibit a natural order. This assessment is misleading. The primary feature of the condition is not about the treatment choices alone, but rather a relationship between a sequence of IV-values whereby higher rankings of the z-values associated with higher rankings of the counterfactual choices $T_i(z)$ for each agent i. This relationship is best understood in terms of sequences of IV-values and treatment choices:

Ordered Monotonicity Condition (OMC): There exist an ordered sequence of treatment statuses $t_1 < \cdots < t_{N_T}$ in $\mathcal{T}$ and a sequence of IV-values $(z_1, \ldots, z_{N_Z})$ in $\mathcal{Z}$ such that $T_i(z_1) \leq \cdots \leq T_i(z_{N_Z})$ holds for each $i \in \mathcal{I}$.

The OMC is a slightly more inclusive version of the Angrist and Imbens (1995) condition. The monotonicity holds whenever it is possible to assign values to treatment choice T such that a sequence of IV-values produces an increasing sequence of counterfactual choices across all types. In the binary choice model, OMC is equivalent to the monotonicity condition of LATE. To gain intuition, consider the case where $T \in \{1, 2, 3\}$, $Z \in \{z_1, z_2, z_3\}$ and Ordered Monotonicity $T_i(z_1) \leq T_i(z_2) \leq T_i(z_3)$ holds. The response matrix that contains all the admissible response types of model is:

$$\boldsymbol{R} = \begin{array}{c} \begin{array}{cccccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 & \boldsymbol{s}_9 & \boldsymbol{s}_{10} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 \\ 1 & 1 & 1 & 2 & 2 & 3 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 2 & 3 & 3 & 2 & 3 & 3 & 3 \end{bmatrix} \end{array} \begin{array}{l} \\ T(z_1) \\ T(z_2) \\ T(z_3) \end{array} \tag{19}$$

The response matrix contains ten types. The matrix satisfies the OMC because the choices weakly increase as we move from one row to another. In the general case of N_T choices and N_Z IV-values, the OMC yields a total of $\binom{N_T+N_Z-1}{N_T-1}$ admissible response types. A choice model is said to be *saturated* with respect to the OMC if it contains all the response types that adhere to OMC. Otherwise stated, it is not possible to add another type without violating the condition. On the other hand, a choice model is said to be *unsaturated* with respect to the OMC when it yields only a subset of the types admitted by OMC. This is the case of example E.3. Note that if we order the IV-values to (z_1, z_0, z_2) and assign the choice values $t_1 = 1, t_0 = 2, t_2 = 3$, we obtain the following response matrix representation:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ z_1 \\ z_0 \\ z_2 \end{array}, \quad \boldsymbol{R} = \begin{array}{c} \begin{array}{cccccccc} \boldsymbol{s}_2 & \boldsymbol{s}_1 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 \end{array} \\ \begin{bmatrix} 1 & 2 & 3 & 1 & 2 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 & 2 & 2 & 1 & 3 \\ 1 & 2 & 3 & 3 & 3 & 2 & 3 & 3 \end{bmatrix} \end{array} \begin{array}{l} \\ T(z_1) \\ T(z_0) \\ T(z_2) \end{array} \tag{20}$$

The choice model adheres to OMC because the treatment values weakly increase as we move from one row to another. This implies that $T_i(z_1) \leq T_i(z_0) \leq T_i(z_2)$ holds for all $i \in \mathcal{I}$ and therefore OMC holds. The choice model is unsaturated since it does not contain all the ten possible types. This means that the choice restrictions that characterize the choice model are more restrictive than the OMC alone.

Next we show that the OMC is economically justified by *supermodular incentives*. Specifically, incentives are termed supermodular if there exists a sequence of IV-values $(z_1, ..., z_{N_Z})$ and a sequence of treatment choices $(t_1, ...t_{N_T})$ such that, for all $j = 1, ..., N_Z - 1$, and $k = 1, ..., N_Z - 1$, we have:

$$\textbf{Supermodular Incentives: } \boldsymbol{L}[z_{k+1}, t_j] - \boldsymbol{L}[z_k, t_j] \leq \boldsymbol{L}[z_{k+1}, t_{j+1}] - \boldsymbol{L}[z_k, t_{j+1}]. \tag{21}$$

Supermodular incentives imply that the difference in incentives across IV-values weakly increases as we progress through the sequence of treatment choices. This pattern includes choice incentives that weakly increase towards higher ranks of IV-values and higher treatment statuses. We term an incentive matrix $\boldsymbol{L}$ *strictly supermodular* if the inequality in (21) is strictly enforced. We can now state the following result:

Theorem T.2. For any IV model with N_T choices and N_Z IV-values that satisfy Assumptions (1)–(3) and the Choice Rule (7), Ordered Monotonicity holds if and only if incentives are supermodular. Moreover, strictly supermodular incentives generate saturated response matrices.

Proof. See Appendix B.5. □

Theorem **T.2** states that supermodular incentives ensure the OMC. For notational convenience, it is useful to define $\Delta\boldsymbol{L}$ as the row-difference of an incentive matrix $\boldsymbol{L}$:

$$\Delta\boldsymbol{L}[k,j] = (\boldsymbol{L}[z_{k+1}, t_j] - \boldsymbol{L}[z_k, t_j]);\ \ k = 1, .., N_Z - 1;\ j = 1, ..., N_T.$$

Incentives are supermodular if $\Delta\boldsymbol{L}$ is weakly increasing in both row and column dimensions. In addition, the incentives are strictly supermodular the columns of $\Delta\boldsymbol{L}$ are strictly increasing, namely, $\Delta\boldsymbol{L}[\cdot, j] < \Delta\boldsymbol{L}[\cdot, j+1]$. Examples of supermodular incentives for $T \in \{1, 2, 3\}$ and $Z = \in \{z_1, z_2, z_3\}$ are:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \Rightarrow \Delta\boldsymbol{L} = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 1 & 5 \end{bmatrix} \tag{22}$$

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \Rightarrow \Delta\boldsymbol{L} = \begin{bmatrix} -1/2 & 0 & 1/2 \\ -1/2 & 0 & 1/2 \end{bmatrix} \tag{23}$$

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \Rightarrow \Delta\boldsymbol{L} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{24}$$

The first example (22) presents a Vandermonde matrix which exhibits increasing choice incentives in both row and the column dimensions. The matrix satisfies strictly supermodularity since the columns of the difference matrix $\Delta\boldsymbol{L}$ is strictly increasing. According to **T.2**, these incentives yield the saturated response matrix displayed in (19).

The second example (23) displays a choice pattern in which z_1 offers full incentives for choice

1; z_3 offers full incentives for choice 3; while z_2 splits the incentives between choices 1 and 3. Strictly supermodularity also holds since the columns of $\Delta\boldsymbol{L}$ are strictly increasing. According to **T.2**, these incentives also yield the saturated response matrix displayed in (19).

The final incentive matrix (23) refers to the example in equation (20). It satisfies supermodularity since the columns in $\Delta\boldsymbol{L}$ are weakling increasing. However, strict supermodularity does not hold as the columns of $\Delta\boldsymbol{L}$ are not strictly increasing. These incentives generate the unsaturated response matrix in (20). The generated response satisfies the OMC and has fewer types than the saturated version. It is worth noting that the fewer types yield additional identification power. It also means that the incentive matrix yields choice restrictions that subsume and outperform the OMC.

3.2 Investigating Unordered Monotonicity

Heckman and Pinto (2018) propose an Unordered Monotonicity Condition that applies to treatment choices that are not ordered. The condition states that for each pair of IV-values $(z, z') \in \mathcal{Z}^2$ and for each $t \in \mathcal{T}$,

$$\mathbf{1}[T_i(z) = t] \leq \mathbf{1}[T_i(z') = t] \; \forall \, i \text{ or } \mathbf{1}[T_i(z) = t] \leq \mathbf{1}[T_i(z') \leq t] \; \forall \, i. \tag{25}$$

The condition means that for each of the choices t, an IV-change must induce induces all agents towards t or all agents away from t. Heckman and Pinto (2018) show that the condition naturally arises in a range of IV settings in which treatment choices do not have a clear ordering structure. They also show that unordered monotonicity enables us to express the indicator for choice t as a latent threshold indicator akin to the result in Vytlacil (2002). Specifically, $\mathbf{1}[T(z) = t] = \mathbf{1}[P_t(z) \geq U_t]$, where $P_t(z) = P(T = t|Z = z)$ is the propensity score and $U_t \sim Unif[0, 1]$ is an unobserved random variable with uniform distribution in $[0, 1]$ that is statistically independent of Z.[10] Pinto (2022) explores this choice representation to evaluate the Moving to Opportunity Intervention as described in example **E.4**. Finally, the monotonicity condition can be equivalently stated in terms of IV-sequences:

Unordered Monotonicity Condition (UMC): For each choice t, there exists a sequence of IV-values $(z_1^{(t)}, ..., z_{N_Z}^{(t)})$ in $\mathcal{Z}$ such that $\mathbf{1}[T_i(z_1^{(t)}) = t] \leq ... \leq \mathbf{1}[T_i(z_{N_Z}^{(t)}) = t]$.

UMC posits that for each treatment choice t, there is a sequence of the IV-values $(z_1^{(t)}, ..., z_{N_Z}^{(t)})$ for which higher rankings of this IV-sequence induce agents towards choosing t, that is, $\mathbf{1}[T_i(z_j^{(t)}) = t] \leq \mathbf{1}[T_i(z_{j+1}^{(t)}) = t]$ for $j = 1..., N - 1$. Note that the IV sequence can and do differ across choices $t \in \mathcal{T}$. This is in contrast with the OMC, which posits that there exists a *single sequence* of IV values that induces all agents to choose higher treatment values. As mentioned, both conditions collapse to the monotonicity condition of Imbens and Angrist (1994) in the case of a binary choice. In the case of multiple choices, the UMC does not imply the OMC and vice-versa.

[10] Heckman and Pinto (2018) assume a general model where choice $T = f(Z, \boldsymbol{V})$ is a function of the instrument Z and an absolutely continuous unobserved random vector $\boldsymbol{V}$ that is statistically independent of Z.

In practical terms, UMC means that it is possible to reorder the rows and columns of the response matrix to generate a lower triangular matrix with respect to each choice t. We revisit the example E.4 to illustrate this property:

$$\boldsymbol{R} = \begin{array}{cc} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 \end{matrix} & \\ \begin{bmatrix} t_h & t_m & t_l & t_h & t_h & t_m & t_h \\ t_h & t_m & t_l & t_m & t_l & t_m & t_m \\ t_h & t_m & t_l & t_l & t_l & t_l & t_h \end{bmatrix} & \begin{matrix} T(z_c) \\ T(z_8) \\ T(z_e) \end{matrix} \end{array}, \quad \boldsymbol{R}_h = \begin{array}{cc} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_7 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_6 \end{matrix} & \\ \begin{bmatrix} t_h & t_m & t_m & t_l & t_m & t_l & t_m \\ t_h & t_h & t_l & t_l & t_m & t_l & t_l \\ t_h & t_h & t_h & t_h & t_m & t_l & t_m \end{bmatrix} & \begin{matrix} T(z_8) \\ T(z_e) \\ T(z_c) \end{matrix} \end{array}, \tag{26}$$

$$\boldsymbol{R}_m = \begin{array}{cc} \begin{matrix} \boldsymbol{s}_2 & \boldsymbol{s}_6 & \boldsymbol{s}_4 & \boldsymbol{s}_7 & \boldsymbol{s}_1 & \boldsymbol{s}_3 & \boldsymbol{s}_5 \end{matrix} & \\ \begin{bmatrix} t_m & t_l & t_l & t_h & t_h & t_l & t_l \\ t_m & t_m & t_h & t_h & t_h & t_l & t_h \\ t_m & t_m & t_m & t_m & t_h & t_l & t_l \end{bmatrix} & \begin{matrix} T(z_e) \\ T(z_c) \\ T(z_8) \end{matrix} \end{array}, \quad \boldsymbol{R}_l = \begin{array}{cc} \begin{matrix} \boldsymbol{s}_3 & \boldsymbol{s}_5 & \boldsymbol{s}_4 & \boldsymbol{s}_6 & \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_7 \end{matrix} & \\ \begin{bmatrix} t_l & t_h & t_h & t_m & t_h & t_m & t_h \\ t_l & t_l & t_m & t_m & t_h & t_m & t_m \\ t_l & t_l & t_l & t_l & t_h & t_m & t_h \end{bmatrix} & \begin{matrix} T(z_c) \\ T(z_8) \\ T(z_e) \end{matrix} \end{array}. \tag{27}$$

The matrix $\boldsymbol{R}$ is the original response matrix of example E.4. Matrix $\boldsymbol{R}_h$ rearranges the columns and rows of the original matrix to generate a lower triangular matrix w.r.t. t_h. This ordering reveals that the number of types taking value t_h increase as we move along the IV-sequence z_8, z_e, z_c.[11] This means that the IV-sequence z_8, z_e, z_c induce agents to choose t_h and the following inequality holds:

$$\mathbf{1}[T_i(z_e) = t_h] \leq \mathbf{1}[T_i(z_8) = t_h] \leq \mathbf{1}[T_i(z_c) = t_h] \, \forall \, i \in \mathcal{I}.$$

Matrices $\boldsymbol{R}_m$ and $\boldsymbol{R}_l$ show that it is also possible to generate lower triangular matrices w.r.t. t_m and t_l via row and column permutations. Consequently, UMC is satisfied. The response matrix is said to be saturated w.r.t. the UMC because it is not possible to add another response type without violating the condition.

In contrast, OMC does not hold for the response matrix $\boldsymbol{R}$ in (26) since it is not possible to assign values to treatment choices t_h, t_m, t_l that ensure increasing sequences of counterfactuals $T_i(z_c) \leq T_i(z_8) \leq T_i(z_e)$ across all types. For instance, selecting choice values such that $t_h < t_m < t_l$ results in an increasing sequence of treatment values for response type $\boldsymbol{s}_4 = [t_h, t_m, t_l]'$, but it fails to generate an increasing sequence for the type $\boldsymbol{s}_7 = [t_h, t_m, t_h]'$.

A simple criterion to verify if a response matrix satisfies the UMC is to check if the matrix does not contain a 2×2 submatrix in which the diagonal contains a choice t and the off-diagonal does not.[12] This prohibit pattern prevents us to transform the response matrix into a lower triangular matrix of t-values as illustrated in equations (26)–(27). For instance, response matrix (20) satisfies OMC, but it does not satisfy UMC. In fact, the 2×2 submatrix formed by columns $\boldsymbol{s}_5, \boldsymbol{s}_6$, and rows z_1, z_3, has choice 2 in its diagonal but lacks choice 2 in its off-diagonal, which violates UMC.

We introduce the concept of *Monotonic incentives*, which is pivotal in analyzing the economic incentives underlying the UMC. An incentive matrix is termed t-monotonic if for any two IV-values

[11] Under z_8, only $\boldsymbol{s}_1$ takes the value t_h. Under z_e, the types $\boldsymbol{s}_1$ and $\boldsymbol{s}_7$ take the value t_h, and under z_c, the types that take the value t_h are $\boldsymbol{s}_1, \boldsymbol{s}_7, \boldsymbol{s}_4, \boldsymbol{s}_5$.

[12] See Heckman and Pinto (2018) for a discussion on this property.

z, z' we have that:

$$\textbf{t-Monotonic Incentives:}\ \boldsymbol{L}[z',t]-\boldsymbol{L}[z,t] \leq \boldsymbol{L}[z',t']-\boldsymbol{L}[z,t']\ \forall\, t' \in \mathcal{T}$$
$$\text{or } \boldsymbol{L}[z',t]-\boldsymbol{L}[z,t] \geq \boldsymbol{L}[z',t']-\boldsymbol{L}[z,t']\ \forall\, t' \in \mathcal{T}. \tag{28}$$

Incentives are t-monotonic if, given any instrumental change, the difference in the choice incentives for t is either the maximum among all treatment choices or the minimum among all the treatment choices. The next theorem describes relevant properties of t-monotonic incentives (28):

Theorem T.3. Let the IV model described by Assumptions (1)–(3) whose choice incentives are given by an incentive matrix $\boldsymbol{L}$ that satisfies Choice Rule (7). If incentives are t-monotone for a choice $t \in \mathcal{T}$, then:
(i) $\mathbf{1}[T_i(z)=t] \leq \mathbf{1}[T_i(z')=t]\ \forall\, i$ or $\mathbf{1}[T_i(z)=t] \geq \mathbf{1}[T_i(z')=t]\ \forall\, i$ for any $z, z' \in \mathcal{Z}$.
(ii) UMC holds if and only if $\boldsymbol{L}$ is t-monotonic for all $t \in \mathcal{T}$.

Proof. See Appendix B.7. □

Item *(i)* of the theorem states that t-monotonic incentives imply a monotonicity condition on the choice indicator $\mathbf{1}[T_i(z)=t]$, namely, a change in the instrument induce all agents towards choice t or all agents against the choice. Item *(ii)* states that for UMC to hold, t-monotonic incentives must apply to all choices. To illustrate this concept, let's consider the analysis of the incentive matrix in the Moving to Opportunity (MTO) example **E.4**:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} t_h & t_m & t_l \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_c \\ z_8 \\ z_e \end{array} \quad \therefore \quad \begin{array}{r|ccc} & t_h & t_m & t_l \\ \hline \boldsymbol{L}[z_8,t]-\boldsymbol{L}[z_c,t] & 0 & 1 & 1 \\ \boldsymbol{L}[z_e,t]-\boldsymbol{L}[z_c,t] & 0 & 1 & 0 \\ \boldsymbol{L}[z_8,t]-\boldsymbol{L}[z_e,t] & 0 & 0 & 1 \end{array}$$

The second matrix above displays the incentive difference $\boldsymbol{L}[z',t]-\boldsymbol{L}[z,t]$ for $(z',z) \in (z_8,z_c),(z_e,z_c),(z_8,z_e)$ across all three choices of $t \in t_h, t_m, t_l$. Note that the incentive differences take values one or zero. The incentive matrix is t_h-monotonic since the difference for t_h is zero for each IV-combination and it is the smallest value of the incentive differences. The incentive matrix is also t_m-monotonic because the incentive difference $\boldsymbol{L}[z',t_m]-\boldsymbol{L}[z,t_m]$ is one for IV-values $(z_8,z_c),(z_e,z_c)$, and therefore the maximum across all choices, and zero for IV-values (z_8,z_e), therefore the minimum across all choices. A similar analysis applies to choice t_l. Thus, t-monotonicity holds for all choices and, as expected, the incentive matrix generates a model in which UMC holds.

Indeed, UMC holds for all binary incentive matrices whose incentive differences for IV values $(z.z')$ take only two values across the treatment choices. These include the binary incentive matrices in which an IV-change induces a monotonicity change of incentives across all choices, that is:

$$\boldsymbol{L}[z,t] \in \{0,1\} \text{ such that } z,z' \in \mathcal{Z},\ \boldsymbol{L}[z',t] \leq \boldsymbol{L}[z,t]\ \forall\, t \in \mathcal{T} \text{ or } \boldsymbol{L}[z,t] \leq \boldsymbol{L}[z',t]\ \forall\, t \in \mathcal{T}. \tag{29}$$

Incentive pattern (29) means that an IV-change will either weakly increase the choice incentives across all treatment choices or weakly decrease the choice incentives across all treatment choices. It is straightforward to demonstrate that this incentive pattern satisfies Item (ii) of Theorem **T.3**. In the case of $Z \in \{z_1, z_2, z_3\}$ and $T \in \{1,2,3\}$, there are four non-equivalent matrices that satisfy (29):

$$\boldsymbol{L} = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array}, \quad \boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array}, \quad \boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array}, \quad \boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array}.$$

Note that the first matrix above is equivalent to the incentive matrix of the MTO example **E.4**. These four matrices generates a distinct response matrices satisfying the UMC.

To gain deeper understanding of Theorem **T.3**, let us examine the incentive matrix of example **E.3**. The incentive matrix and its corresponding incentive differences across IV-values are presented below:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} t_0 & t_1 & t_2 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}\begin{array}{c} \\ z_0 \\ z_1 \\ z_2 \end{array} \therefore \begin{array}{c|ccc} & t_0 & t_1 & t_2 \\ \hline \boldsymbol{L}[z_1,t] - \boldsymbol{L}[z_0,t] & 0 & 1 & 0 \\ \boldsymbol{L}[z_2,t] - \boldsymbol{L}[z_1,t] & 0 & \text{-1} & 1 \\ \boldsymbol{L}[z_2,t] - \boldsymbol{L}[z_0,t] & 0 & 0 & 1 \end{array}$$

The incentive matrix fails to be t_0-monotone because the incentive difference between z_2 and z_1 at t_0, $\boldsymbol{L}[z_2,t_0] - \boldsymbol{L}[z_1,t_0] = 0$, is neither the minimum, $\boldsymbol{L}[z_2,t_1] - \boldsymbol{L}[z_1,t_1] = -1$, nor the maximum, $\boldsymbol{L}[z_2,t_2] - \boldsymbol{L}[z_1,t_2] = 1$, across the choices. This implies that the monotonicity on the choice indicator $\mathbf{1}[T(z) = t_0]$ does not hold and therefore UMC is not satisfied. We can corroborate this fact by checking for a prohibit pattern of choice t_0 in the response matrix (13). Indeed, the 2×2 sub-matrix comprising types $\boldsymbol{s}_5, \boldsymbol{s}_6$ and IV-values z_1, z_2 contains t_0 in the diagonal but does not contain $t - 0$ in the off-diagonal.[13]

A simple method to check for t-monotonicity in binary incentive matrices is to split the incentive matrix $\boldsymbol{L}$ into $\boldsymbol{L}_t^0$ and $\boldsymbol{L}_t^1$ such that $\boldsymbol{L}_t^0$ contains the z-rows $\boldsymbol{L}[z,\cdot]$ such that $\boldsymbol{L}[z,t] = 0$ and $\boldsymbol{L}_t^1$ contains the z-rows such that $\boldsymbol{L}[z,t] = 1$. Under this notation, we can present the following result:

Corollary C.1. Let $\boldsymbol{L}$ be a binary incentive matrix that satisfies Choice Rule (7), then incentives are t-monotonic for $t \in \mathcal{T}$ if and only if matrices $\boldsymbol{L}_t^1$ and $\boldsymbol{L}_t^0$ are lonesum.

Proof. See Appendix B.8. □

The corollary states that a necessary and sufficient condition for t-monotonicity to hold in a binary incentive matrix $\boldsymbol{L}$ is that matrices $\boldsymbol{L}_t^1$ and $\boldsymbol{L}_t^0$ are lonesum matrices. This means that $\boldsymbol{L}_t^1$ and $\boldsymbol{L}_t^0$ can be transformed into lower triangular matrices via row and column permutations.[14]

[13]Similar analysis applies to the incentive matrices (22) and (23). These incentives are not t-monotonic for choice 2 since we observe $\boldsymbol{L}[z_2,1] - \boldsymbol{L}[z_1,1] < \boldsymbol{L}[z_2,2] - \boldsymbol{L}[z_1,2] < \boldsymbol{L}[z_2,3] - \boldsymbol{L}[z_1,3]$. These incentive matrices (22) and (23) yield the saturated OMC response matrix in (19), but UMC does not hold. Indeed, the 2×2 submatrix of columns $\boldsymbol{s}_2, \boldsymbol{s}_5$ and rows z_2, z_3 contain choice 2 in its off-diagonal but no choice 2 in its diagonal.

[14]See (Ryser, 1957) for a study on the properties of lonesum matrices.

This is also equivalent to state that no 2×2 submatrices in $\boldsymbol{L}_t^1$ or $\boldsymbol{L}_t^0$ take the prohibit pattern of numbers 1 in one diagonal and numbers 0 in the other diagonal. For instance, consider the incentive matrix of example E.5:

$$
\boldsymbol{L} = \begin{array}{c c}
\begin{array}{ccc} 0 & 2 & 4 \end{array} & (z_2, z_4) \\
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} & \begin{array}{c} (0,0) \\ (0,1) \\ (1,0) \\ (1,1) \end{array}
\end{array}
\therefore
\begin{array}{c}
\boldsymbol{L}_2^0 = \begin{array}{c c}
\begin{array}{ccc} 0 & 2 & 4 \end{array} & (z_2, z_4) \\
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} (0,0) \\ (0,1) \end{array}
\end{array} \\[2ex]
\boldsymbol{L}_2^1 = \begin{array}{c c}
\begin{array}{ccc} 0 & 2 & 4 \end{array} & (z_2, z_4) \\
\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} & \begin{array}{c} (1,0) \\ (1,1) \end{array}
\end{array}
\end{array},
\begin{array}{c}
\boldsymbol{L}_4^0 = \begin{array}{c c}
\begin{array}{ccc} 0 & 2 & 4 \end{array} & (z_2, z_4) \\
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} (0,0) \\ (1,0) \end{array}
\end{array} \\[2ex]
\boldsymbol{L}_4^1 = \begin{array}{c c}
\begin{array}{ccc} 0 & 2 & 4 \end{array} & (z_2, z_4) \\
\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} & \begin{array}{c} (0,1) \\ (1,1) \end{array}
\end{array}
\end{array}.
$$

The first matrix displays the incentive matrix $\boldsymbol{L}$ of example E.5. Matrices $\boldsymbol{L}_2^0$ and $\boldsymbol{L}_2^1$ split $\boldsymbol{L}$ according to the incentives of choice 2. These matrices do not contain the prohibit pattern (2×2 identity matrix). Thus, these incentives are t-monotonic regarding choice 2. Matrices $\boldsymbol{L}_4^0$ and $\boldsymbol{L}_4^1$ refer to choice 4. These matrices do not present the prohibit pattern either. Therefore, the incentives are also t-monotonic for choice 4. Incentives are zero for choice 0 (first column), thus $\boldsymbol{L}_0^0 = \boldsymbol{L}$, and the matrix displays the prohibit pattern in the columns associated to choices 2 and 4, and rows $(0,1)$ and $(1,0)$. Therefore conditions for UMC are not met.

We can further investigate the model since the t-monotonicity properties of the incentive matrix imply monotonicity conditions on the choices themselves. The incentive matrix is t-monotonic for choices 2 and 4. Thus, the monotonicity condition (28) holds for each of these choices and there must exist IV-sequences that induce each of these choices. Indeed, we can reorder the rows and columns of the response matrix to review a progressive and monotonic pattern of choice selection:

$$
\boldsymbol{R}_2 = \begin{array}{c c}
\begin{array}{ccccccccc} \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_2 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_8 & \boldsymbol{s}_1 & \boldsymbol{s}_3 & \boldsymbol{s}_9 \end{array} & \\
\begin{bmatrix} 2 & 4 & 0 & 4 & 4 & 4 & 0 & 4 & 4 \\ 2 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 4 \\ 2 & 2 & 2 & 2 & 4 & 4 & 0 & 4 & 4 \\ 2 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 4 \end{bmatrix} & \begin{array}{c} T(0,1) \\ T(0,0) \\ T(1,1) \\ T(1,0) \end{array}
\end{array}
$$

$$
\boldsymbol{R}_4 = \begin{array}{c c}
\begin{array}{ccccccccc} \boldsymbol{s}_9 & \boldsymbol{s}_8 & \boldsymbol{s}_3 & \boldsymbol{s}_5 & \boldsymbol{s}_4 & \boldsymbol{s}_7 & \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_6 \end{array} & \\
\begin{bmatrix} 4 & 2 & 0 & 2 & 2 & 2 & 0 & 2 & 2 \\ 4 & 4 & 0 & 0 & 0 & 2 & 0 & 0 & 2 \\ 4 & 4 & 4 & 4 & 2 & 2 & 0 & 2 & 2 \\ 4 & 4 & 4 & 4 & 4 & 4 & 0 & 0 & 2 \end{bmatrix} & \begin{array}{c} T(0,1) \\ T(0,0) \\ T(1,1) \\ T(0,1) \end{array}
\end{array}
$$

The equations above show that the response matrix can be reordered into a lower triangular matrix with respect to choices 2 and 4. Thus the monotonicity condition (28) holds for choices 2 and 4. The same feature does not apply to choice 0 since the 2×2 submatrix of response types $\boldsymbol{s}_2$ and $\boldsymbol{s}_3$ and rows $T(0,1)$ and $T(1,0)$ display the prohibit pattern, namely, it has choice 0 in the diagonal, but does not contain choice 0 in the off-diagonal. As expected, UMC does not hold. OMC does not hold either since no IV-sequence yields a weakly increasing sequence of treatment choices across all types. Finally, we can use the propensity score representation of the t-monotonicity conditions to express the choice model by the following structural equations:

$$T = \begin{cases} 0 & \text{if } P_2(Z) < U_2 \text{ and } P_4(Z) < U_4, \\ 2 & \text{if } P_2(Z) \geq U_2, \\ 4 & \text{if } P_4(Z) \geq U_4, \end{cases}$$

where $P_t(Z) \equiv P(T = t|Z)$, $U_t \sim Unif[0, 1]$, and $Z \perp\!\!\!\perp U_t$ for $t \in \{2, 4\}$. This structural representation arises from the t-monotonicity of choices 2 and 4, and the fact that choice 0 is the complement of choices 2 and 4. This representation allows us to express the counterfactual outcomes as functions of propensity scores, that is, $Y(2)$ is a function of the propensity score $P_2(Z)$, $Y(4)$ is a function of $P_4(Z)$, and $Y(0)$ is a function of both propensity scores. Additional identification power emerges when assuming functional forms for these counterfactuals or exploring baseline variables to generate variation in propensity scores. In the case of continuous instruments, this structural representation can be used to identify average treatment effects using the framework proposed by Lee and Salanié (2018).

In summary, our analyses show that the incentive matrix of example **E.3** generate a choice model that satisfies only OMC. The choice incentives of example **E.4** generates a model satisfying only UMC. Finally, the incentives of example **E.5** generate a choice model in which either UMC or OMC holds. Next section explores incentives that lead to choice models where both UMC and OMC hold.

3.3 Incentives that Justify Recoding Treatment into an Exposure Indicator

The empirical analysis of IV models frequently involves the conversion of a multi-valued treatment into a binary variable that indicates exposure to a treatment. A typical example is to recode years of schooling into a dummy variable for college or high school graduation.[15] Angrist and Imbens (1995) argue that recoding the treatment status is problematic since the common 2SLS estimand recovers a weighted average of effects that usually do not have the intended causal interpretation. This problem has been recently studied by Andresen and Huber (2021) and Rose and Shem-Tov (2023). A simple example clarifies this issue.

Consider the IV model where $T \in \{0, 2, 4\}$ denotes years of college education. Let $Z \in \{z_0, z_1\}$, be an instrument where z_1 offers increasing incentives to greater years of college education while z_0 is a baseline comparison that offers no choice incentives:

[15] Numerous empirical studies undertake a binary conversion of a multi-valued treatment, including Aizer and Doyle (2015); Arteaga (2021); Bhuller et al. (2020); Black et al. (2005); Carneiro et al. (2011); Finkelstein et al. (2012); Kane and Rouse (1995); Mogstad and Wiswall (2016).

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 0 & 2 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \\ z_0 \\ z_1 \end{array} \quad \therefore \quad \boldsymbol{R} = \begin{array}{c} \begin{array}{cccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 \end{array} \\ \begin{bmatrix} 0 & 2 & 4 & 0 & 0 & 2 \\ 0 & 2 & 4 & 2 & 4 & 4 \end{bmatrix} \end{array} \begin{array}{c} \\ T(z_0) \\ T(z_1) \end{array}$$

The incentive matrix above satisfies strict supermodularity, which yields a saturated response matrix with respect to OMC. The corresponding response matrix has six types $\boldsymbol{s}_1$–$\boldsymbol{s}_6$ ensuring that $T_i(z_0) \leq T_i(z_1)$ holds for all $i \in \mathcal{I}$. Types $\boldsymbol{s}_1$–$\boldsymbol{s}_3$ are always-takers, while $\boldsymbol{s}_4 - \boldsymbol{s}_6$ are compliers. A research intends to evaluate the causal effect of four-year college graduation on an outcome of interest. Thus he recodes the treatment T into the binary variable $D = \mathbf{1}[T = 4]$ that indicates if the agent has completed a four-year college education. The Wald estimand of the 2SLS regression recovers the following average causal response:

$$\frac{E(Y|Z=z_1) - E(Y|Z=z_0)}{E(D|Z=z_1) - E(D|Z=z_0)} = \underbrace{\frac{E(Y(4)-Y(0)|\boldsymbol{s}_5)P(\boldsymbol{s}_5) + E(Y(4)-Y(2)|\boldsymbol{s}_6)P(\boldsymbol{s}_6)}{P(\boldsymbol{S} \in \{\boldsymbol{s}_5, \boldsymbol{s}_6\})}}_{\text{Intended Effect (extra-margin)}} + \underbrace{\frac{E(Y(2)-Y(0)|\boldsymbol{s}_4)P(\boldsymbol{s}_4)}{P(\boldsymbol{S} \in \{\boldsymbol{s}_5, \boldsymbol{s}_6\})}}_{\text{Unintended Effect (intra-margin)}}$$

This estimand presents two problems. First, it conflates an intended effect with an unintended one. The intended effect is the weighted average of the causal effect of four-year college graduation against no college, $E(Y(4) - Y(0)|\boldsymbol{s}_5)$, and two-year college, $E(Y(4) - Y(2)|\boldsymbol{s}_6)$. Types $\boldsymbol{s}_5$ and $\boldsymbol{s}_6$ display an extra-margin variation of the treatment T since D changes from zero to one as T shifts from 0 or 2 to 4. Conversely, the unintended effect evaluates the causal effect of two-year college versus no college, $E(Y(2) - Y(0)|\boldsymbol{s}_4)$. Response type $\boldsymbol{s}_4$ displays an intra-margin variation, as D remains constant when T changes from 0 to 2. The second problem, highlighted by Andresen and Huber (2021), is that the binary treatment violates the IV exclusion restriction since the IV affects the counterfactual outcomes through channels beyond D. A solution to both problems is to prevent intra-margin treatment variation by eliminating type $\boldsymbol{s}_4$.

Rose and Shem-Tov (2021) coined the term Extensive Margin Compliers Only (EMCO) for a monotonicity condition that prevents intra-margin treatment variation in IV models with a binary instrument. We present a revised condition that extends their approach to the case of a categorical instrument. For a given treatment status $t \in \mathcal{T}$ and any for any $z, z' \in \text{supp}(Z)$ we have that:

$$t\textbf{-EMCO:} \quad \mathbf{1}[T_i(z) = t] \leq \mathbf{1}[T_i(z') = t] \forall\, i \text{ or } \mathbf{1}[T_i(z) = t] \geq \mathbf{1}[T_i(z') = t] \forall\, i \tag{30}$$

$$\text{and } T_i(z) \neq T_i(z') \Rightarrow T_i(z) = t \text{ or } T_i(z') = t. \tag{31}$$

The t-EMCO condition refers to a single treatment status t and contains two requirements: a monotonicity condition of the choice indicator (30), and a no intra-margin condition (31), which ensures that any choice shift within each complier must be confined between t and one other choice. The following response matrix displays the response types for $T \in \{0, 2, 4\}$ and $Z \in \{z_1, z_2, z_3\}$, when 4-EMCO holds:

$$\boldsymbol{R} = \begin{array}{c} \begin{array}{ccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 \end{array} \\ \begin{bmatrix} 0 & 2 & 4 & 0 & 2 & 0 & 2 \\ 0 & 2 & 4 & 4 & 4 & 0 & 2 \\ 0 & 2 & 4 & 4 & 4 & 4 & 4 \end{bmatrix} \end{array} \begin{array}{c} \\ T(z_1) \\ T(z_2) \\ T(z_3) \end{array} \tag{32}$$

Response types $\boldsymbol{s}_1 - \boldsymbol{s}_3$ are always-takes. The remaining types $\boldsymbol{s}_6 - \boldsymbol{s}_7$ are the compliers. The monotonicity of the choice indicator holds since $\mathbf{1}[T_i(z_1) = 4] \leq \mathbf{1}[T_i(z_2) = 4] \leq \mathbf{1}[T_i(z_3) = 4]$. Compliers do not display intra-margin variation. The counterfactual choices vary only between two treatment statuses: choices 4 or 0 in $\boldsymbol{s}_4$ and $\boldsymbol{s}_6$, and choices 4 or 2 in $\boldsymbol{s}_5$ and $\boldsymbol{s}_7$. This feature ensures that the Wald estimand evaluates a weighted average of intended treatment effects.

Theorem T.4. *t-EMCO implies OMC and UMC.*

Proof. See Appendix B.10. □

The t-EMCO is a particular case of monotonicity condition that satisfies both OMC and UMC. Indeed, a saturated response matrix with respect to EMCO is also saturated with respect to UMC, but unsaturated with respect to OMC. Results associated with OMC and UMC apply. In particular, the 2SLS estimand evaluates a weighted average of per-unit treatment effects that compares choice t with the remaining choices across compliers (Angrist and Imbens, 1995), and each choice can be expressed by a separable equation on the propensity score and a latent variable (Heckman and Pinto, 2018).

An incentive matrix $\boldsymbol{L}$ is said to satisfy the t-EMCO incentives when the following condition applies:

$$t\textbf{-EMCO Incentives}: \quad \boldsymbol{L}[z,t] = \boldsymbol{L}[z',t] \; \forall z, z' \in \text{supp}(Z), \tag{33}$$

$$\text{and } \; \boldsymbol{L}[z,t'] = \boldsymbol{L}[z,t''] \; \forall t', t'' \in \mathcal{T} \setminus \{t\} \text{ and } z \in \text{supp}(Z). \tag{34}$$

This condition states that incentives for choice t are constant *across* all IV values, while the incentives for the remaining choices are the same for any *given* IV-value. We can now state the following result:

Theorem T.5. *Let the IV model described by Assumptions (1)–(3) whose choice incentives are given by an incentive matrix $\boldsymbol{L}$ where Choice Rule (7) holds. If $\boldsymbol{L}$ satisfy the t-EMCO incentives (33), then t-EMCO (30) holds.*

Proof. See Appendix B.11. □

Consider the IV example of this section where $T \in \{0, 2, 4\}$ and $Z \in \{z_1, z_2, z_3\}$. Examples of 4-EMCO incentives that generate the response matrix in (32) are:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 0 & 2 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 1/2 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ z_3 \\ z_2 \\ z_1 \end{array}, \quad \boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 0 & 2 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_3 \\ z_2 \\ z_1 \end{array}, \quad \boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 0 & 2 & 4 \end{array} \\ \begin{bmatrix} -1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_3 \\ z_2 \\ z_1 \end{array}$$

The key feature of these incentive matrices is that the incentives for choice 4 remain constant among IV values while the incentives for choices 2 and 4 are the same given each IV-value. It is easy to verify that EMCO incentives are monotonic and supermodular, which yield a response matrix that jointly satisfy OMC and UMC.

4 An Empirical Exercise

The Unite States hosts approximately 12 million undocumented residents, with nearly half originating from Mexico. According to Borjas (1987), migration decisions are positively influenced by wage differentials between origin and destination countries but are offset by migration costs. Their research posits a negative selection in migration patterns since lower-skilled workers who can absorb the migration costs benefit the most from relocating to the US. This perspective is further corroborated by Angelucci (2015). She finds that Oportunidades, Mexico's paramount anti-poverty initiative, spurred the emigration of lower-skilled, undocumented migrants to the US.

Behrman et al. (2005) emphasize that Oportunidades' impact on schooling attendance boosts basic English proficiency and analytical competencies, both pivotal skills for success in the US labor market. Their analysis posits a non-monotonic relationship between education and migration: attaining fundamental skills heightens the propensity to migrate. Yet, further human capital accumulation diminishes this likelihood, as it renders the domestic labor market more appealing relative to its international counterpart. This pattern is also supported by Chiquiar and Hanson (2005); Hanson (2006).

To elaborate, the Mexican education system is structured into three stages:

1. Primaria or Elementary School (Grades 1-6),
2. Secundaria or Middle School (Grades 7-9), and
3. Preparatoria or High School (Grades 10-12).

Fundamental English skills are introduced during Secundaria, as detailed in Table 1. Consequently, completing Secundaria is expected to exerts a positive influence on migration decisions, while advancing from Secundaria to Preparatoria (or completing high school) is anticipated to have a negative effect on migration.

We use a decade of panel data from the Oportunidades Program to examine the impact of scrolling on migration patterns. Oportunidades is a pioneering conditional cash transfer program in Mexico. The program was launched in 1997 and it randomly assigned 505 rural villages to either a treatment group (320 villages) or a control group (185 villages). Families in the treated villages received bi-monthly cash transfers, which often amounted to 20% to 30% of their household income. The transfer was contingent upon their school-age children attend school. Households in control villages had to wait for two years before receiving these benefits (Gertler, 2004).

Our study employs panel data covering the period from 1997 to 2007 to assess the influence of Oportunidades on U.S. migration among individuals who were 12 to 13 years old in 1997. This age group comprises the participants most affected by the differential schooling incentives between the treated and control groups. [16]

The sample comprises more than 3,000 individuals residing in impoverished rural areas. Schooling data were collected in 2003, and we utilize census data from 1997, 2003, and 2007 to examine migration patterns. Approximately 18.0% of males and 10.3% of females experienced migration, with the majority making the journey to the United States between the ages of 16 and 22. Table 5 presents a statistical overview of baseline variables categorized by gender. As anticipated, baseline variables exhibit a balanced distribution across randomization arms, and none of the differences in means between the assignment groups achieve statistical significance.

Following our previous notation, we use $Z \in \{z_0, z_1\}$ for the randomization arms, T for years schooling, and Y for the migration outcome. Figure 1 displays the distribution of schooling at onset of the intervention in 1997 and six years after the intervention, in 2003. It is evident that Oportunidades promotes schooling. Thus, a common approach to modeling such interventions is to assume OMC, specifically, $T_i(z_0) \le T_i(z_1)$ for all $i \in \mathcal{I}$. The OMC provides a justification for employing 2SLS regressions to examine the causal effect of Treatment T on the migration outcome Y. The results from this well-known methodology are presented in Table 3.

In our sample, Oportunidades significantly increased migration patterns and schooling attainment among males. Specifically, the intervention raised migration rates by approximately 3.0 percentage points for males. This represents an 22% increase compared to the migration probability of control group males. In the realm of education, Oportunidades led to an increase of roughly one-fourth of a school year. Table 3 also showcases the 2SLS regression where the random assignment of Oportunidades acts as an IV to evaluate the impact of education on migration. The estimated coefficient for males is around 0.060 and is statistically significant at 10% significance level.

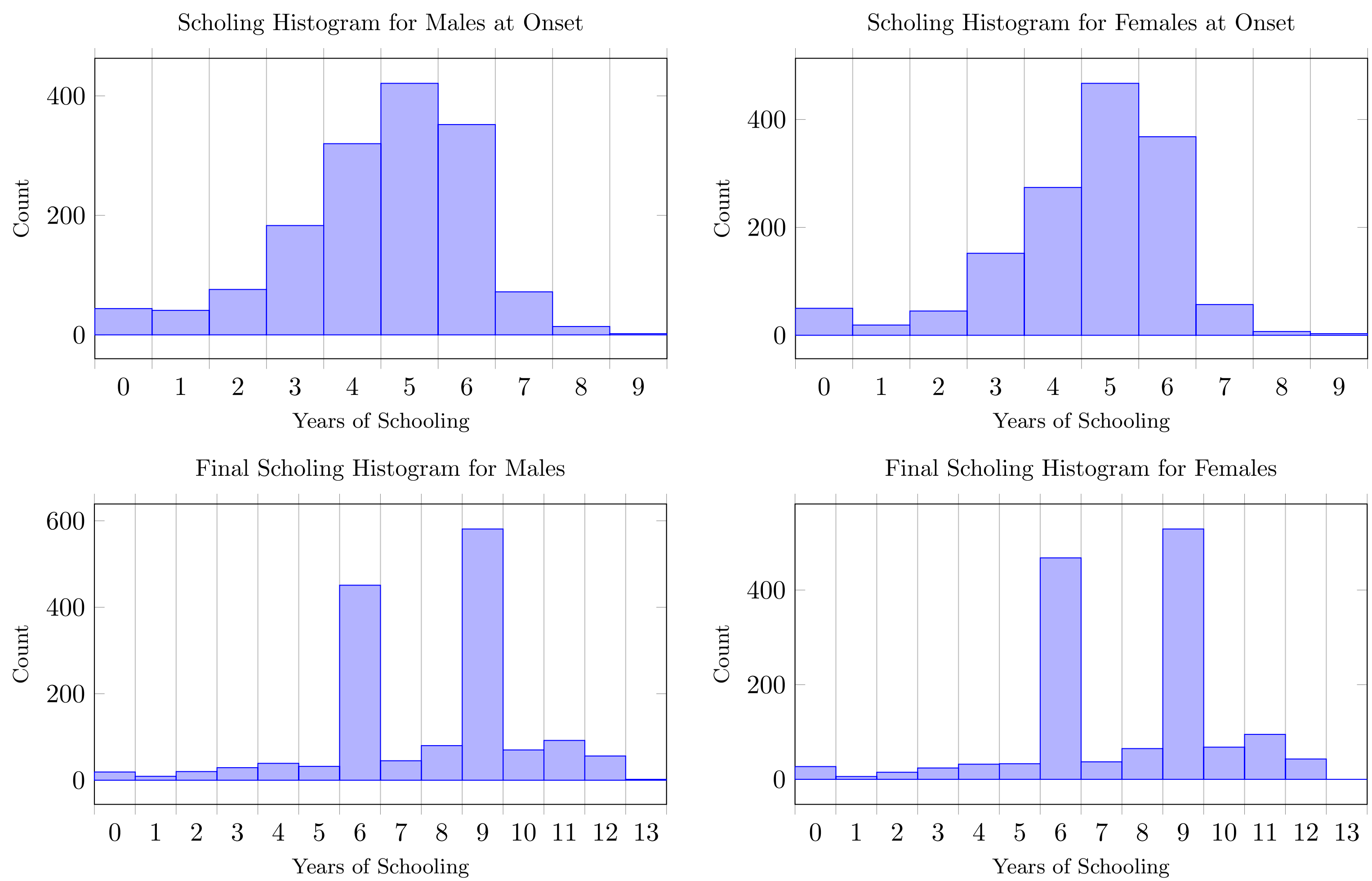
The 2SLS analysis is useful to assess the overall impact of education on migration. However, coefficient evaluates a weighted average per-unit treatment effect across all individuals that increase their education when the instrument shifts from z_0 to z_1. It is difficult to relate this causal interpretation with the migration questions we seek to address. In order to advance, we evaluate a stylized model that benefits from the choice incentives and the observed patterns of school choices.

4.1 Stylized Model

We devise a stylized model that explores the tendency for education choices to cluster predominantly around the completion of Secundaria (9 years of schooling), as shown in Figure 1. Recall that completing Secundaria is a milestone in the analysis of the impact of schooling on migration since

[16]The age range was also defined in accordance to two criteria: (1) the lower boundary is set high enough to ensure that the schooling survey in 2003 measures the final schooling attainment; and (2) the upper boundary is set low enough to include individuals who were 22 years old in 2007 when the migration data was collected.

Figure 1: Histograms of Years of Schooling at the Onset and After the Intervention



This histogram displays the observed distribution of years of schooling at the onset of the intervention in 1997 and six years later, in 2003. The sample includes participants who were between the ages of 11 and 12 years old at the start of the intervention.

basic English skills are taught during this stage. Thus we transform the schooling variable into the index $T \in \{1,2,3\}$ where $T=1$ stand for schooling less than Secundaria, and $T=2$ stand for Secundaria competition, $T=3$ stand for schooling beyond Secundaria.

Participants in our selected age group face three main schooling choices: (1) whether to continue studying in the year of the intervention; (2) whether to complete Secundaria; and (3) whether to continue studying beyond Secundaria. The treated group received cash transfers during all school years, which provided incentives to complete Secundaria and additional incentives to continue further studies. In contrast, the participants of the control group did not receive cash transfers during the initial years of Secundaria, influencing their decision. However, the control students who chose to continue their education received cash transfers a few years later, influencing their choice to study beyond Secundaria. The Incentive matrix corresponding to these choice incentives and its associated response matrix are presented below:

$$\boldsymbol{L} = \begin{array}{cc} \begin{array}{ccc} 1 & 2 & 3 \end{array} & \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} & \begin{array}{c} z_0 \\ z_1 \end{array} \end{array} \therefore \ \boldsymbol{R} = \begin{array}{cc} \begin{array}{ccccc} \boldsymbol{s}_{11} & \boldsymbol{s}_{22} & \boldsymbol{s}_{33} & \boldsymbol{s}_{12} & \boldsymbol{s}_{13} \end{array} & \\ \begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 2 & 3 \end{bmatrix} & \begin{array}{c} T(z_0) \\ T(z_1) \end{array} \end{array}. \tag{35}$$

The response matrix is obtained by applying Choice Rule (7) to the incentive matrix. It contains three always-takers $\boldsymbol{s}_{11}, \boldsymbol{s}_{22}, \boldsymbol{s}_{33}$, and two compliers $\boldsymbol{s}_{12}, \boldsymbol{s}_{13}$. The incentives are t-monotonic for all choices, leading to a saturated response matrix w.r.t. UMC. The incentives are also supermodular. As a result, OMC holds, but the matrix is not saturated w.r.t. OMC since choice restrictions eliminate type $[2,3]'$.

The identification analysis stems from Theorem **T.1**. All type probabilities are (just) identified. Always-taker probabilities are given by:[17]

$P(\boldsymbol{s}_{11}|X) = P(T=1|z_1,X)$, $P(\boldsymbol{s}_{22}|X) = P(T=2|z_0,X)$, and $P(\boldsymbol{s}_{33}|X) = P(T=3|z_0,X)$.

The probabilities for compliers are identified by:

$P(\boldsymbol{s}_{12}|X) = P(T=2|z_1,X) - P(T=2|z_0,X)$, and $P(\boldsymbol{s}_{22}|X) = P(T=3|z_1,X) - P(T=3|z_0,X)$.

There are six counterfactual outcomes that are identified. Counterfactual outcomes for always-takers are given by:

$E(Y(1)|\boldsymbol{s}_{11}) = E(Y|T=1,z_1,X), E(Y(2)|\boldsymbol{s}_{22}) = E(Y|T=2,z_0,X)$ and $E(Y(3)|\boldsymbol{s}_{33}) = E(Y|T=3,z_0,X)$.

The remaining counterfactuals are identified as LATE-type parameters:

$$\begin{aligned} E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\}, X) &= LATE_X(\mathbf{1}[T=1]), \\ E(Y(2)|\boldsymbol{S} = \boldsymbol{s}_{12}, X) &= LATE_X(\mathbf{1}[T=2]), \\ E(Y(3)|\boldsymbol{S} = \boldsymbol{s}_{13}, X) &= LATE_X(\mathbf{1}[T=3]), \\ \text{where: } LATE_X(W) &\equiv \frac{E(Y \cdot W|Z=z_1,X) - E(Y \cdot W|Z=z_0,X)}{E(W|Z=z_1,X) - E(W|Z=z_0,X)}. \end{aligned}$$

We are most interested in two causal effects: $E(Y(2) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$, which is the causal effect

[17] We use $P(\boldsymbol{s}|X)$ and $P(T=t|z,X)$ as short-hand notation for $P(\boldsymbol{S}=\boldsymbol{s}|X)$ and $P(T=t|Z=z,X)$ respectively.

of completing Secundaria on migration, and $E(Y(3) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$, which is the causal effect of studying beyond Secundaria. Unfortunately, we cannot disentangle $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\})$ into $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$ without additional assumptions. We solve this problem of partial identification by invoking the assumption of comparable compliers:[18]

$$\text{Comparable Compliers:} \quad Y(1) \perp\!\!\!\perp \boldsymbol{S}|(T(z_0) \neq T(z_1), X). \tag{36}$$

The assumption states, that, conditioned on the compliers and on the baseline variables X, the counterfactual outcome $Y(1)$ is independent of the types. Effectively, this assumption enables the point-identification of the model by equalizing the counterfactual means for $Y(1)$ among compliers, $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12}, X) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13}, X)$.

4.2 Estimating Type Probabilities

We devise a doubly robust estimator that employs machine learning techniques to evaluate causal parameters. The method stems from the work of Navjeevan, Pinto, and Santos (2023) and has desirable properties commonly shared by this type of estimator. The method yields asymptotically normal estimators that guarantees double robustness against misspecification (Robins et al., 1995) and possesses the mixed bias property in (Chernozhukov et al., 2018). The method also benefits from variety of plug-in machine learning techniques as described in Smucler et al. (2019), Chernozhukov et al. (2022), and Chernozhukov et al. (2022).

To gain intuition, we examine the identification of type probabilities in greater detail. Let $\boldsymbol{P}_{T|X}(t) \equiv [P(T = t|Z = z_0, X), P(T = t|Z = z_1, X)]'$ be the 2×1 vector of choice probabilities across IV-values, and $\boldsymbol{P}_{T|X} \equiv [\boldsymbol{P}_{T|X}(1)', \boldsymbol{P}_{T|X}(2)', \boldsymbol{P}_{T|X}(3)']'$ be the 6×1 vector of propensity scores. Moreover, the 5×1 vector of type probabilities conditioned on X is:

$$\boldsymbol{P}_{\boldsymbol{S}|X} = [P(\boldsymbol{s}_{11}|X), P(\boldsymbol{s}_{22}|X), P(\boldsymbol{s}_{33}|X), P(\boldsymbol{s}_{12}|X), P(\boldsymbol{s}_{13}|X)]'.$$

These vectors are related by the equation $\boldsymbol{P}_{T|X} = \boldsymbol{B}\boldsymbol{P}_{\boldsymbol{S}|X}$, where $\boldsymbol{B} \equiv [\boldsymbol{B}_1', \boldsymbol{B}_2', \boldsymbol{B}_3']'$ is the 8×5 binary matrix that stacks the indicator matrices $\boldsymbol{B}_t = \mathbf{1}[\boldsymbol{R} = t]$ across the treatment choices. The response matrix $\boldsymbol{R}$ is defined in (35). In this notation, we can express each of the type probabilities as a linear combination of the propensity scores:

$$P(\boldsymbol{S} = \boldsymbol{s}|X) = \boldsymbol{\nu}_{\boldsymbol{s}}\boldsymbol{P}_{Z|X} \text{ such that } \boldsymbol{\nu}_{\boldsymbol{s}} \equiv \ell_{\boldsymbol{s}}'(\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'. \tag{37}$$

The term $\boldsymbol{\nu}_{\boldsymbol{s}}$ is primary in our analysis. It is a known 6×1 vector defined as $\ell_{\boldsymbol{s}}'(\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'$, where $\ell_{\boldsymbol{s}}$ is a 5×1 canonic vector that takes value one for type $\boldsymbol{s}$ and zero otherwise. Vector $\boldsymbol{\nu}_{\boldsymbol{s}}$ can be understood as a map $\nu_{\boldsymbol{s}}(z, t)$ from the support of (Z, T) to $\mathbb{R}$. In this notation, we can rewrite equation (37) as:

$$P(\boldsymbol{S} = \boldsymbol{s}|X) = \sum_{t \in \mathcal{T}} \sum_{z \in \mathcal{Z}} \nu_{\boldsymbol{s}}(z, t) P(T = t|Z = z, X). \tag{38}$$

[18] For examples of works that invoke this assumption, see, for instance, Mountjoy (2022); Navjeevan et al. (2023).

To construct the doubly robust estimator, we represent the type probability as the expectation of a function κ such that $P(\boldsymbol{S}=\boldsymbol{s}) = E(\kappa_{\boldsymbol{s}}(T,Z,X))$.[19] The doubly robust estimator is based on the following the orthogonal score representation of type probabilities:

$$P(\boldsymbol{S} \ = \boldsymbol{s}) = \sum_{t\in\mathcal{T}} E_{ZX}\left(\kappa_{\boldsymbol{s}}(t,Z,X)\cdot\left(\mathbf{1}[T=t]-P(T=t|Z,X)\right)\right) + \sum_{t\in\mathcal{T}} E_X\left(\sum_{z\in\mathcal{Z}} \nu_{\boldsymbol{s}}(z,t)P(T=t|Z=z,X)\right),$$

where $E_{ZX}(\cdot)$ is an expectation over the joint distribution of (Z,X) and $E_X(\cdot)$ is an expectation over X. The identifying moment condition has two nuisance parameters, the function $\kappa_{\boldsymbol{s}}(t,Z,X)$ and the propensity score $P(T=t|Z,X)$. We assess these nuances via plug-in estimators, that is, we evaluate the propensity score $P(T=t|Z,X)$ by $\boldsymbol{h}(Z,X)\boldsymbol{\beta}_t$, and the kappa function $\kappa_{\boldsymbol{s}}(t,Z,X)$ by $\boldsymbol{h}(Z,X)\boldsymbol{\gamma}_{\boldsymbol{s},t}$, where $\boldsymbol{\beta}_t, \boldsymbol{\gamma}_t$ are p-dimensional linear coefficients and $\boldsymbol{h}(Z,X) = [b_1(Z,X),...,b_p(Z,X)]'$ denotes a p-dimensional vector of function of (Z,X) including all the pairwise interactions of these variables. In our application, $\boldsymbol{h}(Z,X)$ comprises X, Z, and their interaction. Specifically, our estimator is obtained from the following algorithm:

Step 1. Partition the sample index $\mathcal{I} = \{1,\ldots,n\}$ into K subsets such that $\cup_{k=1}^{K}\{\mathcal{I}_k\} = \mathcal{I}$, where the number of partitions K is commonly fixed to five. Let $\mathcal{I}_k^c = \mathcal{I}\setminus\mathcal{I}_k$ be the complement of $\mathcal{I}_k$

Step 2. For each value $t\in\{1,2,3\}$ and each partition k, compute the estimator $\hat{\gamma}_{t,k,\boldsymbol{s}}$ associated with the kappa function $\kappa_{\boldsymbol{s}}(t,Z,X)$ by minimizing the following expression:

$$\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k} \in \arg\min_{\boldsymbol{\gamma}\in\mathbf{R}^p} \sum_{i\in\mathcal{I}_k^c}\left(\frac{1}{2}(\boldsymbol{h}(Z_i,X_i)'\boldsymbol{\gamma})^2 + \sum_{z\in\mathcal{Z}}\nu_{\boldsymbol{s}}(t,z)\boldsymbol{h}(z,X_i)'\boldsymbol{\gamma}\right) + \alpha_{\boldsymbol{\gamma}}\|\boldsymbol{\gamma}\|_1, \tag{39}$$

where $\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k}$ is evaluated using all data that is not in $\mathcal{I}_k$, while $\alpha_{\boldsymbol{\gamma}}$ is the penalty parameter determined by a cross-validation procedure employing all sampling data.

Step 3. For each value $t\in\{1,2,3\}$ and each partition I_k, compute the estimator $\hat{\boldsymbol{\beta}}_{t,k}$ associated with the propensity score $P(T=t|Z,X)$ via the least absolute shrinkage and selection operator (lasso) procedure that minimizes the following expression:

$$\hat{\boldsymbol{\beta}}_{t,k} \in \arg\min_{\boldsymbol{\beta}\in\mathbf{R}^p} \sum_{i\in\mathcal{I}_k^c}(1[T_i=t]-\boldsymbol{h}(Z_i,X_i)'\boldsymbol{\beta})^2 + \alpha_{\boldsymbol{\beta}}\|\boldsymbol{\beta}\|_1,$$

where $\alpha_{\boldsymbol{\beta}}$ is the penalty parameter also determined by via cross-validation procedure.[20]

Step 4. Given $\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k}$ and $\hat{\boldsymbol{\beta}}_{t,k}$, we compute the orthogonal score estimator $\hat{\psi}_{\boldsymbol{s},i,k}$ for each participant $i\in\mathcal{I}_k$ and for each partition k :

$$\hat{\psi}_{\boldsymbol{s},k,i} \equiv \sum_{t\in\mathcal{T}}\left(\boldsymbol{h}(Z_i,X_i)'\hat{\gamma}_{\boldsymbol{s},t,k}\cdot\left(1[T_i=t]-\boldsymbol{h}(Z_i,X_i)'\hat{\boldsymbol{\beta}}_{t,k}\right) + \sum_{z\in\mathcal{Z}}\nu_{\boldsymbol{s}}(t,z)\boldsymbol{h}(z,X_i)'\hat{\boldsymbol{\beta}}_{t,k}\right).$$

[19] See Navjeevan, Pinto, and Santos (2023) for a in-depth discussion of the rationale of this approach.

[20] Note that the penalty parameters $\alpha_{\boldsymbol{\beta}}$ and $\alpha_{\boldsymbol{\gamma}}$ do not need to be the same, but the functions $\boldsymbol{h}(Z,X)$ are the same in steps 2 and 3.

Step 5. The estimator for the propensity score $P(\boldsymbol{S} = \boldsymbol{s})$ is the average of the orthogonal scores within partition, that is, $\hat{\psi}_{\boldsymbol{s},k} = |\mathcal{I}_k|^{-1} \sum_{i \in \mathcal{I}_k} \sum_{t \in \mathcal{T}} \hat{\psi}_{\boldsymbol{s},k,i}$. The final estimate is the average of the orthogonal scores across partitions, namely, $\hat{\psi}_{\boldsymbol{s}} = n^{-1} \sum_{k=1}^{K} \hat{\psi}_{\boldsymbol{s},k} \cdot |\mathcal{I}_k|$.

Step 6. Inference is performed via the bootstrap multiplier method. For each partition k, we draw B samples $\{W_i^{(b)}\}_{i \in \mathcal{I}_k}$ of i.i.d. standard normals to compute:

$$\hat{\psi}_{\boldsymbol{s},k}^{(b)} = \hat{\psi}_{\boldsymbol{s},k} + \frac{1}{n} \sum_{i \in \mathcal{I}_k}^{n} W_i^{(b)} \left(\hat{\psi}_{\boldsymbol{s},k,i} - \hat{\psi}_{\boldsymbol{s},k} \right), \text{ and } \hat{\psi}_{\boldsymbol{s}}^{(b)} = n^{-1} \sum_{k=1}^{K} \hat{\psi}_{\boldsymbol{s},k}^{(b)} \cdot |\mathcal{I}_k|.$$

We use the distribution of $\hat{\psi}_{\boldsymbol{s}}^{(b)}$ to compute the standard error of the estimator for the type probability.

A few notes on the estimation method are in order. The sample splitting in Step 1 is not necessary to secure normality of the estimator and can be voided. The estimators in Steps 2 and 3 allow for some degree of flexibility. In our setup, $(Z, X)'\hat{\boldsymbol{\beta}}_{t,k}$ estimates the propensity score and $\boldsymbol{h}(Z, X)'\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k}$ estimates the kappa function. These estimates can be obtained by suitable alternative machine learning estimators. For instance, it is possible to transform the minimization that evaluates $\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k}$ in Step 2 into a standard lasso-type estimator.

Let $\boldsymbol{H}_k(z) \equiv \boldsymbol{h}(z, \boldsymbol{X})$ denotes the $|\mathcal{I}_k^c| \times p$ matrices that stack $\boldsymbol{h}(z, X_i)'$ across participants $i \in \mathcal{I}_k^c$. In the same token, let $\boldsymbol{H}_k \equiv \boldsymbol{h}(\boldsymbol{Z}, \boldsymbol{X})$ be the matrix that stakes $\boldsymbol{h}(Z_i, X_i)'$ across $i \in \mathcal{I}_k^c$, and let $\boldsymbol{\iota}_k$ be the $|\mathcal{I}_k^c|$-dimensional vector of ones. In this notation, the minimization of Step 2 can be equivalently expressed as:[21]

$$\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k} \in \arg\min_{\boldsymbol{\gamma} \in \mathbf{R}^p} \sum_{i \in \mathcal{I}_k^c} (\boldsymbol{h}(Z_i, X_i)'\boldsymbol{\theta} - \boldsymbol{h}(Z_i, X_i)'\boldsymbol{\gamma})^2 + \alpha_{\boldsymbol{\gamma}} \|\boldsymbol{\gamma}\|_1,$$

$$\text{where } \boldsymbol{\theta} \equiv (\boldsymbol{H}_k'\boldsymbol{H}_k)^{-1} \left(\sum_{z \in \mathcal{Z}} \nu_{\boldsymbol{s}}(t, z) \boldsymbol{H}_k(z)'\boldsymbol{\iota}_k \right).$$

The term $\boldsymbol{h}(Z_i, X_i)'\boldsymbol{\theta}$ can be roughly understood as the projection of the function $\sum_{z \in \mathcal{Z}} \nu_{\boldsymbol{s}}(t, z)\boldsymbol{h}(z, X_i)$ into the space generated by $\boldsymbol{h}(Z_i, X_i)$. Finally, we use the leave-one-out sampling scheme in all cross-validation methods.

In Table 4, we present the estimated probabilities for each type. The aggregate probability for the always-takers is approximately 0.90, indicating that 90% of the sample comprises students who persist with their schooling choice towards Secundaria irrespective of their allocation to either treatment or control groups. The probability for type $\boldsymbol{s}_{11}$ stands at around 0.43, suggesting that nearly half of the sample consists of students who do not complete Secundaria, regardless of the incentives from Oportunidades. The probability associated with type $\boldsymbol{s}_{22}$ is close to 0.34, denoting that a third of the sample consistently chooses to finalize their Secundaria. Lastly, the probability for type $\boldsymbol{s}_{33}$ is approximately 0.13, implying that a mere 13% of the students opt to pursue education beyond Secundaria, irrespective of receiving the Oportunidades incentives or not.

[21]The estimator is numerically equivalent to evaluating the minimum of the function in Step 2. The equivalence is easy be shown when expressing the minimization using matrix notation.

The sum of the probabilities for compliers, $\boldsymbol{s}_{12}$ and $\boldsymbol{s}_{13}$, totals 0.093. This means that about 9% of the students change their choice towards completing Secundaria when the incentives provided by Oportunidades are available. The majority of these students, about 7%, consists of participants of type $\boldsymbol{s}_{12}$ who shift for not completing Secundaria to completing Secundaria. A smaller share of sample, about 2%, comprises compliers that change their student decision from not completing Secundaria when assigned to control to studying beyond Secondaria when assigned to the treatment.

4.3 Estimating Causal Effects

We now describe the doubly robust estimators used to evaluate the counterfactual outcomes and the causal effects of our model. Our discussion mirrors the approach we took when investigating type probabilities. To provide more insight, we examine the identification of counterfactual outcomes through moment conditions.

Let $\boldsymbol{E}_{Y|X}(t) \equiv [E(Y \cdot \mathbf{1}[T = t]|Z = z_0, X), E(Y \cdot \mathbf{1}[T = t]|Z = z_1, X)]'$ be the 2×1 vector of conditional outcome moments. As mentioned, $\boldsymbol{B}_t = \mathbf{1}[\boldsymbol{R} = t]$ denotes the binary matrix that indicates which elements in the response matrix $\boldsymbol{R}$ in (35) takes value $t \in \{1, 2, 3\}$. We use this notation to express the identified counterfactual outcomes – $E(Y(1)|\boldsymbol{s}_{11})$, $E(Y(2)|\boldsymbol{s}_{22})$, $E(Y(3)|\boldsymbol{s}_{33})$ $E(Y(2)|\boldsymbol{S} = \boldsymbol{s}_{12}, X)$ and $E(Y(3)|\boldsymbol{S} = \boldsymbol{s}_{13}, X)$ – in the following fashion:

$$E(Y(t)|\boldsymbol{S} = \boldsymbol{s})P(\boldsymbol{S} = \boldsymbol{s}|X) = \boldsymbol{\nu}_{\boldsymbol{s},t}\boldsymbol{E}_{Z|X} \text{ such that } \boldsymbol{\nu}_{\boldsymbol{s},t} \equiv \ell_{\boldsymbol{s}}'\boldsymbol{B}_t'(\boldsymbol{B}_t\boldsymbol{B}_t')^{-1}, \tag{40}$$

where $\ell_{\boldsymbol{s}}$ is a 5×1 canonic vector that indicates type $\boldsymbol{s}$. Similar to our analysis of type probabilities, we can express $\boldsymbol{\nu}_{\boldsymbol{s},t}$ as a function $\nu_{\boldsymbol{s},t}(z)$ from the support of Z to $\mathbb{R}$. In this notation, we can rewrite equation (40) as:

$$E(Y(t)\mathbf{1}[\boldsymbol{S} = \boldsymbol{s}]|X)P(\boldsymbol{S} = \boldsymbol{s}|X) = \sum_{z \in \mathcal{Z}} \nu_{\boldsymbol{s},t}(z)E(Y \cdot \mathbf{1}[T = t]|Z = z, X). \tag{41}$$

It is also worth noting that the response types probability associated with the identified outcome counterfactuals can be identified as:

$$P(\boldsymbol{S} = \boldsymbol{s}|X) = \sum_{z \in \mathcal{Z}} \nu_{\boldsymbol{s},t}(z)E(\mathbf{1}[T = t]|Z = z, X). \tag{42}$$

The doubly robust estimator comprises the joint evaluation of the expectation $E(Y(t)\mathbf{1}[\boldsymbol{S} = \boldsymbol{s}])$ in (41), the probability $P(\boldsymbol{S} = \boldsymbol{s}|X)$ and then taking the ratio of these estimates. Note that both problems are related since they are associated with the same identification function $\nu_{\boldsymbol{s},t}(z)$. The estimator for $E(Y(t)\mathbf{1}[\boldsymbol{S} = \boldsymbol{s}])$ is based on the following orthogonal score:

$$E(Y(t)\mathbf{1}[\boldsymbol{S} = \boldsymbol{s}]) = E_{ZX}\left(Y\kappa_{\boldsymbol{s},t}(Z, X) \cdot \left(Y\mathbf{1}[T = t] - E(Y\mathbf{1}[T = t]|Z, X)\right)\right) + E_X\left(\sum_{z \in \mathcal{Z}} \nu_{\boldsymbol{s},t}(z)E(Y \cdot \mathbf{1}[T = t]|Z = z, X)\right).$$

The function kappa is such that $E(Y\kappa_{\boldsymbol{s},t}(Z, X)) = E(Y(t)\mathbf{1}[\boldsymbol{S} = \boldsymbol{s}])$ and $E(\kappa_{\boldsymbol{s},t}(Z, X)) = P(\boldsymbol{S} =$

$\boldsymbol{s}$). The estimator contains three nuance parameters: the propensity score $P(T=t|Z,X)$ is estimated as by $\boldsymbol{h}(Z,X)\boldsymbol{\beta}_t$, the outcome expectation $E(Y\cdot\mathbf{1}[T=t]|Z=z,X)$ by $\boldsymbol{h}(Z,X)\boldsymbol{\theta}_t$, and the kappa function $\kappa_{\boldsymbol{s},t}(t,Z,X)$ by $\boldsymbol{h}(Z,X)\boldsymbol{\gamma}_{\boldsymbol{s},t}$, where $\boldsymbol{\beta}_t,\boldsymbol{\theta}_t,\boldsymbol{\gamma}_t$ are p-dimensional linear coefficients and $\boldsymbol{h}(Z,X)$ comprises X, Z, and their interaction. The steps of the estimator are closely related to the estimation of type probabilities:

Step 1. Partition $\mathcal{I}$ into $\cup_{k=1}^K\{\mathcal{I}_k\}=\mathcal{I}$, where $\mathcal{I}_k^c=\mathcal{I}\setminus\mathcal{I}_k$.

Step 2. For each k, compute the estimator $\hat{\gamma}_{t,k,\boldsymbol{s}}$ as:

$$\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k}\in\arg\min_{\boldsymbol{\gamma}\in\mathbf{R}^p}\sum_{i\in\mathcal{I}_k^c}\left(\frac{1}{2}(\boldsymbol{h}(Z_i,X_i)'\boldsymbol{\gamma})^2+\sum_{z\in\mathcal{Z}}\nu_{\boldsymbol{s},t}(z)\boldsymbol{h}(z,X_i)'\boldsymbol{\gamma}\right)+\alpha_{\boldsymbol{\gamma}}\|\boldsymbol{\gamma}\|_1, \tag{43}$$

where $\alpha_{\boldsymbol{\gamma}}$ is the penalty parameter determined by a cross-validation (leave-one-out) procedure.

Step 3. For each partition k, compute the estimators $\hat{\boldsymbol{\beta}}_{t,k}$, and $\hat{\boldsymbol{\theta}}_{t,k}$ via lasso:

$$\hat{\boldsymbol{\theta}}_{t,k}\in\arg\min_{\boldsymbol{\theta}\in\mathbf{R}^p}\sum_{i\in\mathcal{I}_k^c}\left(Y\cdot 1[T_i=t]-\boldsymbol{h}(Z_i,X_i)'\boldsymbol{\theta}\right)^2+\alpha_{\boldsymbol{\theta}}\|\boldsymbol{\theta}\|_1,$$
$$\hat{\boldsymbol{\beta}}_{t,k}\in\arg\min_{\boldsymbol{\beta}\in\mathbf{R}^p}\sum_{i\in\mathcal{I}_k^c}\left(1[T_i=t]-\boldsymbol{h}(Z_i,X_i)'\boldsymbol{\beta}\right)^2+\alpha_{\boldsymbol{\beta}}\|\boldsymbol{\beta}\|_1,$$

where $\alpha_{\boldsymbol{\beta}},\alpha_{\boldsymbol{\theta}}$ are the penalty parameters determined by cross-validation.

Step 4. Given $\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k},\hat{\boldsymbol{\beta}}_{t,k}$, and $\hat{\boldsymbol{\theta}}_{t,k}$, for each agent $i\in\mathcal{I}_k$ and each partition k, compute the orthogonal score $\hat{\psi}_{\boldsymbol{s},i,k}$ for $P(\boldsymbol{S}=\boldsymbol{s})$ and $\hat{\varphi}_{\boldsymbol{s},i,k}$ for $E(Y\mathbf{1}[\boldsymbol{S}=\boldsymbol{s}])$

$$\hat{\psi}_{\boldsymbol{s},k,i}\equiv\left(\boldsymbol{h}(Z_i,X_i)'\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k}\cdot\left(1[T_i=t]-\boldsymbol{h}(Z_i,X_i)'\hat{\boldsymbol{\beta}}_{t,k}\right)+\sum_{z\in\mathcal{Z}}\nu_{\boldsymbol{s}}(t,z)\boldsymbol{h}(z,X_i)'\hat{\boldsymbol{\beta}}_{t,k}\right),$$
$$\hat{\varphi}_{\boldsymbol{s},k,i}\equiv\left(\boldsymbol{h}(Z_i,X_i)'\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k}\cdot\left(Y\cdot 1[T_i=t]-\boldsymbol{h}(Z_i,X_i)'\hat{\boldsymbol{\theta}}_{t,k}\right)+\sum_{z\in\mathcal{Z}}\nu_{\boldsymbol{s}}(t,z)\boldsymbol{h}(z,X_i)'\hat{\boldsymbol{\theta}}_{t,k}\right).$$

Step 5. The estimator for $P(\boldsymbol{S}=\boldsymbol{s})$ is the average of the orthogonal scores $\hat{\psi}_{\boldsymbol{s}}=n^{-1}\sum_{k=1}^K\hat{\psi}_{\boldsymbol{s},k}\cdot|\mathcal{I}_k|$, where $\hat{\psi}_{\boldsymbol{s},k}=|\mathcal{I}_k|^{-1}\sum_{i\in\mathcal{I}_k}\sum_{t\in\mathcal{T}}\hat{\psi}_{\boldsymbol{s},k,i}$. The estimator for $E(Y(t)\mathbf{1}[\boldsymbol{S}=\boldsymbol{s}])$ is also the average of the orthogonal scores $\hat{\varphi}_{\boldsymbol{s}}=n^{-1}\sum_{k=1}^K\hat{\varphi}_{\boldsymbol{s},k}\cdot|\mathcal{I}_k|$, where $\hat{\varphi}_{\boldsymbol{s},k}=|\mathcal{I}_k|^{-1}\sum_{i\in\mathcal{I}_k}\sum_{t\in\mathcal{T}}\hat{\varphi}_{\boldsymbol{s},k,i}$. The final estimator for $E(Y(t)|\boldsymbol{S}=\boldsymbol{s})$ is the ratio $\hat{\varphi}_{\boldsymbol{s}}/\hat{\psi}_{\boldsymbol{s}}$.

Step 6. Our inference uses a multiplier bootstrap that draw B samples $\{W_i^{(b)}\}_{i\in\mathcal{I}_k}$ of i.i.d. standard normals for each partition k. We then compute both scores:

$$\hat{\psi}_{\boldsymbol{s},k}^{(b)}=\hat{\psi}_{\boldsymbol{s},k}+\frac{1}{n}\sum_{i\in\mathcal{I}_k}^{n}W_i^{(b)}\left(\hat{\psi}_{\boldsymbol{s},k,i}-\hat{\psi}_{\boldsymbol{s},k}\right),\text{ and }\hat{\psi}_{\boldsymbol{s}}^{(b)}=n^{-1}\sum_{k=1}^K\hat{\psi}_{\boldsymbol{s},k}^{(b)}\cdot|\mathcal{I}_k|,$$
$$\hat{\varphi}_{\boldsymbol{s},k}^{(b)}=\hat{\varphi}_{\boldsymbol{s},k}+\frac{1}{n}\sum_{i\in\mathcal{I}_k}^{n}W_i^{(b)}\left(\hat{\varphi}_{\boldsymbol{s},k,i}-\hat{\varphi}_{\boldsymbol{s},k}\right),\text{ and }\hat{\varphi}_{\boldsymbol{s}}^{(b)}=n^{-1}\sum_{k=1}^K\hat{\varphi}_{\boldsymbol{s},k}^{(b)}\cdot|\mathcal{I}_k|.$$

We use the joint distribution $\{\hat{\psi}_{\boldsymbol{s}}^{(b)}, \hat{\varphi}_{\boldsymbol{s}}^{(b)}\}_{b=1}^{B}$ to estimate the variance matrix of the orthogonal scores denoted by $\hat{\boldsymbol{V}}(\hat{\psi}_{\boldsymbol{s}}, \hat{\varphi}_{\boldsymbol{s}})$. We compute the standard error for the ratio $\hat{\varphi}_{\boldsymbol{s}}/\hat{\psi}_{\boldsymbol{s}}$ using the Delta method, namely, $\hat{\sigma} = \left(n^{-1}\boldsymbol{\omega}'\hat{\boldsymbol{V}}(\hat{\psi}_{\boldsymbol{s}}, \hat{\varphi}_{\boldsymbol{s}})\boldsymbol{\omega}\right)^{1/2}$ where $\boldsymbol{\omega} = [-(\hat{\varphi}_{\boldsymbol{s}}/\hat{\psi}_{\boldsymbol{s}}^2), 1/\hat{\psi}_{\boldsymbol{s}}]'$.

The steps above differ from the estimation of type probabilities in a few instances. Step 2 uses the function $\nu_{\boldsymbol{s},\boldsymbol{t}}(Z)$ instead of $\nu_{\boldsymbol{s}}(T, Z)$. Steps 3 computes an additional parameter $\boldsymbol{\theta}$ while Step 4 computes two orthogonal scores. Steps 5 states that our estimator is a ratio of orthogonal scores means and Step 6 uses bootstrap and the delta method to evaluate the standard error of the ratio.

Recall that we aim to evaluate two causal effects: $E(Y(2) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(2) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$. The procedure outlined above is tailored to estimate any counterfactual outcome that is identified according to the response matrix $\boldsymbol{R}$ (35). These include $E(Y(2)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(3)|\boldsymbol{S} = \boldsymbol{s}_{13})$. The procedure could also be used to evaluate $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\})$, since it is also identified. The procedure however is not suitable to evaluate $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$ separately.

The additional assumption of comparable compliers (36) enable us to disentangle $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\})$ into $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$. The assumption implies that $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\}|X) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12}|X)$ and $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\}|X) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13}|X)$. Note however that this assumption does not imply the unconditional equality $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\}) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12}) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$ because the distribution of baseline variables X may differ across types $\boldsymbol{s}_{12}$ and $\boldsymbol{s}_{13}$. The modification of the procedure is necessary to account for the difference in the distribution of baseline variables X between types. Navjeevan, Pinto, and Santos (2023) solve the same problem in a different setting involving the mediation analysis of a choice model containing seven types. We apply their solution to our setting.

We first consider the task of evaluating $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$. Steps 1, 5 and 6 of the previous procedure remain the same. Steps 2–4 are modified as following.

Step 2'. For each k, compute the estimator $\hat{\gamma}_{t,k,\boldsymbol{s}}$ as:

$$\hat{\boldsymbol{\gamma}}_{1,k} \in \arg\min_{\boldsymbol{\gamma}\in\mathbf{R}^p} \sum_{i\in\mathcal{I}_k^c} \left(\frac{1}{2}(\boldsymbol{h}(Z_i, X_i)'\boldsymbol{\gamma})^2 + -(\boldsymbol{h}(1, X_i)' - \boldsymbol{h}(0, X_i))'\boldsymbol{\gamma}\right) + \alpha_{\boldsymbol{\gamma}}\|\boldsymbol{\gamma}\|_1. \tag{44}$$

Step 3'. For each partition k, compute the parameters $\hat{\boldsymbol{\beta}}_{1,k}$, $\hat{\boldsymbol{\beta}}_{2,k}$, $\hat{\boldsymbol{\theta}}_{1,k}$, $\hat{\boldsymbol{\pi}}_{1,k}$, and $\hat{\boldsymbol{\pi}}_{2,k}$, via the following lasso estimations:

$$\hat{\boldsymbol{\theta}}_{1,k} \in \arg\min_{\boldsymbol{\theta}\in\mathbf{R}^p} \sum_{i\in\mathcal{I}_k^c} \left(Y \cdot 1[T_i = 1] - \boldsymbol{h}(Z_i, X_i)'\boldsymbol{\theta}\right)^2 + \alpha_{\boldsymbol{\theta}}\|\boldsymbol{\theta}\|_1,$$

$$\hat{\boldsymbol{\beta}}_{1,k} \in \arg\min_{\boldsymbol{\beta}\in\mathbf{R}^p} \sum_{i\in\mathcal{I}_k^c} \left(1[T_i = 1] - \boldsymbol{h}(Z_i, X_i)'\boldsymbol{\beta}\right)^2 + \alpha_{\boldsymbol{\beta},1}\|\boldsymbol{\beta}\|_1,$$

$$\hat{\boldsymbol{\beta}}_{2,k} \in \arg\min_{\boldsymbol{\beta}\in\mathbf{R}^p} \sum_{i\in\mathcal{I}_k^c} \left(1[T_i = 2] - \boldsymbol{h}(Z_i, X_i)'\boldsymbol{\beta}\right)^2 + \alpha_{\boldsymbol{\beta},\mathbf{2}}\|\boldsymbol{\beta}\|_1,$$

$$\hat{\boldsymbol{\pi}}_{1,k} \in \arg\min_{\boldsymbol{\pi}\in\mathbf{R}^p} \sum_{i\in\mathcal{I}_k^c} \left(1[T_i = 1]\boldsymbol{h}(Z_i, X_i)'\hat{\gamma}_{1,k} - \boldsymbol{g}(X_i)'\boldsymbol{\pi}\right)^2 + \alpha_{\boldsymbol{\pi},1}\|\boldsymbol{\pi}\|_1,$$

$$\hat{\boldsymbol{\pi}}_{2,k} \in \arg\min_{\boldsymbol{\pi}\in\mathbf{R}^p} \sum_{i\in\mathcal{I}_k^c} \left(1[T_i = 2]\boldsymbol{h}(Z_i, X_i)'\hat{\gamma}_{1,k} - \boldsymbol{f}(X_i)'\boldsymbol{\pi}\right)^2 + \alpha_{\boldsymbol{\pi},2}\|\boldsymbol{\pi}\|_1,$$

where $\boldsymbol{f}(X) \equiv (f_1(X), \ldots, f_q(X))'$ denote a q-dimensional vector of functions of baseline variable.

Step 4'. Given $\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k}, \hat{\boldsymbol{\beta}}_{t,k}$, and $\hat{\boldsymbol{\theta}}_{t,k}$, we can compute the orthogonal score $\hat{\psi}_{\boldsymbol{s},i,k}$ regarding $P(\boldsymbol{S} = \boldsymbol{s}_{21})$ for each agent $i \in \mathcal{I}_k$ and each partition k :

$$\hat{\psi}_{\boldsymbol{s},k,i} \equiv \left(\boldsymbol{h}(Z_i, X_i)'\hat{\boldsymbol{\gamma}}_{\boldsymbol{s},t,k} \cdot \left(1[T_i = 2] - \boldsymbol{h}(Z_i, X_i)'\hat{\boldsymbol{\beta}}_{t,k}\right) + \boldsymbol{h}(1, X_i)' - \boldsymbol{h}(0, X_i)'\hat{\boldsymbol{\beta}}_{t,k}\right).$$

The orthogonal score for $E(Y(1)\mathbf{1}[\boldsymbol{S} = \boldsymbol{s}_{21}])$ is cumbersome. We define the following terms to facilitate notation: $\Theta_i \equiv \boldsymbol{h}(Z_i, X_i)'\hat{\boldsymbol{\theta}}_{1,k}$, $\Lambda_{1,i} \equiv \boldsymbol{h}(Z_i, X_i)'\hat{\boldsymbol{\beta}}_{1,k}$, $\Lambda_{2,i} \equiv \boldsymbol{h}(Z_i, X_i)'\hat{\boldsymbol{\beta}}_{2,k}$, $\Delta_i \equiv \boldsymbol{h}(0, X_i)' - \boldsymbol{h}(1, X_i)$, $\kappa_i \equiv \boldsymbol{h}(Z_i, X_i)'\hat{\gamma}_{1,k}$. $U_i \equiv \boldsymbol{f}(X_i)'\hat{\boldsymbol{\pi}}_{1,k}$, and $C_i \equiv \boldsymbol{f}(X_i)'\hat{\boldsymbol{\pi}}_{2,k}/U_i$. In this notation, we can define the orthogonal score for $E(Y(1)\mathbf{1}[\boldsymbol{S} = \boldsymbol{s}_{21}])$ associated to agent $i \in \mathcal{I}_k$ and each partition k as:

$$\begin{aligned}\hat{\varphi}_{\boldsymbol{s},k,i} \equiv &((Y_i. * 1[T_i = 1] - \Theta_i)\kappa_i + (\Delta_i\Theta_i))C_i - (((1[T_i = 2] - \Lambda_{2,i})\kappa_i) \cdot (\Delta_i\Theta_i))\frac{1}{U_i} \\ &- (((1[T_i = 1] - \Lambda_{1,i})\kappa_i) \cdot (\Delta_i\Theta_i))\frac{C}{U_i} - (((\Delta_i\Lambda_{2,i})) \cdot (\Delta_i\Theta_i))\frac{1}{U_i} - (((\Delta_i\Lambda_{1,i})) \cdot (\Delta_i\Theta_i))\frac{C}{U_i}.\end{aligned}$$

As mentioned, the Steps 5–6 remains the same. This estimator evaluates $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ which enable us to estimate the causal effect $E(Y(2) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ since $E(Y(2)|\boldsymbol{S} = \boldsymbol{s}_{12})$ was already estimated. The standard error of the causal effect is obtained via the multiplier bootstrap. The counterfactual outcome $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$ is obtained by replacing the choice 2 in Steps 3' and Step 4' by the treatment choice 3.

Table 5 presents the causal effects of our model conditioned on different sets of baseline variables. The first panel of the table presents the estimates for $E(Y(2) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$, which evaluates the casual effect of completing Secundaria on migration for the subset of compliers that change from not completing Secundaria to completing it when the incentives of Oportunidades are available. These compliers account for about 7% of the sample. We find that completing Secundaria has a substantial impact on the decision to migrate. The causal effect is about 0.48 and the estimates are statistically significant at 10% significance level.

The second panel of the table displays the estimates for $E(Y(3) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$, which is the causal effect of changing education attainment from not completing Secundaria to study beyond Secundaria on migration. This effect is associated to 2% of the participants. It comprises the subset of compliers that decide to study beyond Secundaria due to the incentives offered by Oportunidades. We find the effect to be negative, relatively small, and not statistically significant. The point estimate of the effect ranges from -0.10 to -0.20 when conditioned on baseline variables.

The main feature of this empirical exercise is the use of incentive analysis to assess the question of whether schooling has a non-monotonic effect on the decision to migrate to the US. We focus on the age range most likely to respond to the schooling incentives offered by Oportunidades. Our stylized model enables us to characterize five types that are driven by economic behavior. We are able to evaluate the share of the participants that do respond to Oprtunidades' incentives and evaluate the causal effects of schooling on migration for those who respond to the incentives. We find compelling evidence that completing Secundaria increases the likelihood of migration. Our key empirical finding however is the difference between the causal effects. While completing Secundaria has a strong effect on the propensity to migrate, studying beyond Secundaria does not. These findings corroborate the hypothesis of several works suggesting a negative and non-monotonic selection of migrants regarding education (Behrman et al., 2005; Borjas, 1987; Chiquiar and Hanson, 2005).

Table 1: Skills taught by School Level
Biggest expected effect on migration moving from primary to secondary

Primaria (Grades 1-6)	Secundaria (grades 7-9)	Preparatoria (grades 10-12)
Basic Reading and writing skills	Intermediate Reading and writing skills	Advanced reading and writing skills
Basic mathematical skills	Intermediate mathematical skills	Advanced mathematical skills
To search for information	Work-oriented skills in workshops	Specialized technical-level skills for work
	(carpentry, plumbing and electricity, cooking, etc)	Critical thinking
To follow instructions	**Essential skills for communication in English**	Vocational guidance for jobs in Mexico
Basic comprehension of natural	Civic and social participation	Social skills
and social environments	Self-learning and Reproductive health	Time management

Table 2: Statistical Description of Baseline Variables

	Males			Females		
	Treated Mean	Control Mean	Diff. Means	Treated Mean	Control Mean	Diff. Means
Age at Onset	11.930	11.929	0.000	11.872	11.909	-0.036
s.e.	0.572	0.553	0.028	0.567	0.584	0.029
Family Speaks Indigenous Language	0.417	0.469	-0.052	0.409	0.425	-0.016
s.e.	0.493	0.499	0.025	0.492	0.495	0.025
Household Assets Index	624.27	616.39	7.880	618.81	625.23	-6.428
s.e.	88.546	98.092	4.685	89.707	91.044	4.542
Number of Household Members	7.581	7.476	0.105	7.625	7.594	0.031
s.e.	2.195	2.097	0.106	2.118	2.049	0.104
Household Members Younger than 17	4.739	4.702	0.037	4.789	4.767	0.022
s.e.	1.784	1.736	0.087	1.705	1.702	0.085
County USA Migration Index	-0.155	-0.170	0.015	-0.115	-0.183	0.068
s.e.	0.843	0.919	0.045	0.878	0.908	0.046
Home Ownership	0.969	0.951	0.018	0.971	0.955	0.016
s.e.	0.174	0.216	0.010	0.167	0.207	0.010
Schooling at Onset	4.546	4.407	0.140	4.535	4.611	-0.076
s.e.	1.618	1.630	0.081	1.577	1.596	0.080
Sample Size	1027	674		1048	645	

This columns of this table presents the statistical description of baseline variables by gender. The first row associated to each variable displays the treatment mean, control mean and the mean difference for males and females. The second row displays the standard deviation of the treated and control means and the standard error for the difference-in-means estimator.

Table 3: Standard 2SLS Analysis on the Effect of Oportunidades on Schooling and Migration

	Males				Females			
	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 1**	**Model 2**	**Model 3**	**Model 4**
Effects of the Oportunidades Intervention								
Migration	**0.037**	**0.033**	0.029	**0.031**	0.005	-0.002	-0.001	0.001
s.e.	0.020	0.019	0.018	0.018	0.016	0.016	0.015	0.015
p-val	0.061	0.073	0.113	0.094	0.753	0.873	0.937	0.965
Schooling	**0.502**	**0.522**	**0.533**	**0.521**	0.022	0.044	0.050	0.076
s.e.	0.121	0.122	0.122	0.122	0.126	0.128	0.128	0.128
p-val	0.000	0.000	0.000	0.000	0.860	0.733	0.695	0.554
Effects of Schooling on Migration								
2SLS	**0.073**	**0.064**	0.055	0.059	0.227	-0.057	-0.024	0.009
s.e.	0.043	0.039	0.037	0.038	1.478	0.390	0.313	0.205
p-val	0.092	0.099	0.135	0.118	0.878	0.885	0.938	0.965
OLS	-0.005	0.000	0.002	0.002	-0.001	-0.001	-0.001	-0.001
s.e.	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003
p-val	0.259	0.980	0.663	0.549	0.803	0.719	0.801	0.792

The table comprises four panels. The first panel displays the causal effects of the Oportunidades on migration. The second panel displays the effects on Schooling (measured in 2003), it is also the first stage for the 2SLS on the third panes, that evaluates the causal effect of Schooling on migration using the Oportunidades random assignment as an instrumental variable for schooling. Each panel presents the estimates by gender across four models that differ in terms of conditioning variables. Model 1 does not use conditioning variables. Model 2 employs age at onset and county migration index. Model 3 adds family characteristics: family members speak indigenous language, number of household members, and number of teenagers. Model 4 includes household assets and house ownership. Estimates consists on the effect, its standard error and the double-sided *p*-value associated with inference that tests if the effect is equal to zero. All estimates are based on the standard OLS and 2SLS regressions. Inference employs clustered errors.

Table 4: Type Probabilities and Causal Effects for Males

Type Probabilities	**Model 1**	**Model 2**	**Model 3**	**Model 4**
$P(\boldsymbol{S} = \boldsymbol{s}_{11})$	**0.436**	**0.436**	**0.435**	**0.436**
s.e.	0.016	0.016	0.016	0.016
p-val	0.000	0.000	0.000	0.000
$P(\boldsymbol{S} = \boldsymbol{s}_{22})$	**0.337**	**0.338**	**0.337**	**0.340**
s.e.	0.019	0.019	0.019	0.019
p-val	0.000	0.000	0.000	0.000
$P(\boldsymbol{S} = \boldsymbol{s}_{33})$	**0.133**	**0.133**	**0.130**	**0.131**
s.e.	0.014	0.014	0.014	0.014
p-val	0.000	0.000	0.000	0.000
$P(\boldsymbol{S} = \boldsymbol{s}_{44})$	**0.071**	**0.071**	**0.070**	**0.068**
s.e.	0.024	0.024	0.025	0.025
p-val	0.004	0.004	0.005	0.007
$P(\boldsymbol{S} = \boldsymbol{s}_{55})$	0.022	0.023	0.026	0.026
s.e.	0.018	0.018	0.018	0.018
p-val	0.206	0.181	0.139	0.149

This table presents the estimates of type probabilities according to the doubly robust orthogonal score estimator described in this section. Estimates are presents for four models that vary in the set of baseline variables X that we seek to condition on. Model 1 does not use baseline variables. Model 2 employs age at onset and county migration index. Model 3 adds family characteristics: family members speak indigenous language, number of household members, and number of teenagers. Model 4 includes household assets and house ownership. Estimates consists on the probability, its standard error and the two-sided p-value associated with inference that tests if the effect is equal to zero. Standard errors are computed using the multiplier bootstrap method.

Table 5: Causal Effects for Males

Causal Effects	**Model 1**	**Model 2**	**Model 3**	**Model 4**
$E(Y(2) - Y(1) \mid \boldsymbol{S} = \boldsymbol{s}_{21})$	**0.480**	**0.487**	**0.472**	**0.503**
s.e.	0.265	0.261	0.261	0.287
p-val	0.070	0.062	0.071	0.079
$E(Y(3) - Y(1) \mid \boldsymbol{S} = \boldsymbol{s}_{31})$	-0.001	-0.118	-0.135	-0.194
s.e.	0.274	0.320	0.310	0.358
p-val	0.998	0.713	0.662	0.587

This table presents the estimates of the causal effects for males. The estimates are obtained according to the doubly robust orthogonal score estimator described in this section. The estimates comprise four models that differ in terms of the set of baseline variables we seek to control for. Model 1 does not include baseline variables. Model 2 employs age at onset and county migration index. Model 3 adds family characteristics: family members speak indigenous language, number of household members, and number of teenagers. Model 4 includes household assets and house ownership. Estimates consists on the effect, its standard error and the two-sided p-value associated with inference that tests if the effect is equal to zero. Standard errors are computed using the multiplier bootstrap method.

5 Summary and Conclusions

This paper has provided a fresh perspective on the identification of causal effects in instrumental variable (IV) models within economics. While the conventional approach has focused on developing novel monotonicity or separability conditions, we have proposed a departure from this mindset.

Our approach is rooted in the utilization of economic incentives and classical economic behavior to identify causal parameters in IV models with multiple choices and categorical instruments. We introduced a flexible framework based on revealed preference analysis, which translates choice incentives into identification conditions. This method has several notable advantages, most notably its independence from statistical or functional form assumptions. Instead, identification conditions arise organically from fundamental economic principles applied to choice incentives, enhancing both credibility and understanding.

Moreover, our framework is versatile enough to accommodate a wide range of non-trivial identification assumptions, making it applicable in scenarios where traditional IV assumptions may not hold. We have demonstrated its flexibility by examining well-established examples of choice incentives in the policy evaluation literature, showcasing its adaptability to real-world empirical research.

We employ our analytical framework to investigate the migration patterns of impoverished Mexican households to the US. A substantial literature on migration investigates the relationship between education attainment and the likelihood of migration. Seminal work of Borjas (1987, 1994) suggests a negative selection in which those with lowest education benefit the most from moving to the US. ON the other hand, Behrman et al. (2005); Chiquiar and Hanson (2005); Hanson (2006) posits a non-monotonic relationship between education and migration, where the fundamental skills such as basic English proficiency taught in Secundaria (middle school) increase the propensity to migrate while additional education reduces migration.

We utilize data from Oportunidades, the largest and most significant anti-poverty program in Mexico, to examine whether schooling has a non-monotonic impact on the decision to migrate to the United States. Employing our incentive framework, we identify two causal effects of education on migration for students responding to the schooling incentives provided by Oportunidades.

Specifically, we assess the impact of completing Secundaria and the effects of pursuing education beyond this level. Our findings provide compelling evidence that completing Secundaria increases the likelihood of migration, whereas advancing schooling beyond middle school has a negative effect on migration. We estimate our model using novel machine learning techniques that assure double robustness of our estimates.

In the broader context of economic research, this paper contributes to the growing body of literature that leverages revealed preference analysis to enhance the identification of causal effects in IV models. Our approach offers a valuable tool for economists grappling with identification issues in diverse and non-standard empirical settings. We make the case that combining economic incentives and classical behavior strengthen the foundations of IV analysis and empowers researchers with a useful tool to evaluate such models.

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