The Economics of Monotonicity Conditions: Exploring Choice Incentives in IV Models

Moshe Buchinsky, Paul Gertler, and Rodrigo Pinto *

January 2, 2024

Abstract

This paper explores the use of economic incentives to identify treatment effects in IV models with multiple choices. We devise a general framework that uses revealed preference analysis to translate choice incentives into identification conditions. We demonstrate that popular identification strategies that rely on monotonicity conditions can be attributed to specific properties of choice incentives. We also show that novel identification strategies emerge when individuals face non-standard choice incentives. We use the framework to revisit key economic studies in the literature of policy evaluation. Finally, we apply the framework to study the impact of education on the migration decisions of low-income Mexican families using data from the Oportunidades program. We employ machine learning techniques to find that completing middle school has a positive effect on migration while additional schooling does not. Our findings contribute to a substantial literature that postulates a non-monotonic relationship between education attainment and the migration decisions of disadvantaged Mexican households.
Keywords: Revealed Preferences, Causal Inference, Identification, Instrumental Variables, Policy Evaluation.
JEL codes: H43, I18, I38. J38.

*Moshe Buchinsky, Paul Gertler and Rodrigo Pinto work at the economics departments of Sciences Po, Berkley Haas and UCLA respectively. We are grateful for the interaction with James Heckman, Steven Durlauf, and Magne Mostag. We also thank many Professors for useful suggestions. Among those, we cite: Andres Santos, Manu Navjeevan, Azeem Shaikh, Manuel Arrelano, Felipe Gonçalves, Thiery Mayer, Jeanne Commault, Andrew Chesher, Michele Pellizzari, Aleksey Tetenov, Giacomo de Giorgi, Federica Braccioli, Yotam Shem-Tov, and Dmitry Arkhangelsky. We also benefit from useful comments on presentations at Getulio Vargas Foundation (EPGE), Princeton, Northwestern University, University of Geneve, UCLA, University of Chicago (Economics and Public Policy Departments), University of Southern California (Schaeffer and CSSR Institutes), and Yale University. Errors are our own.

1 Introduction

Economists have long employed instrumental variables (IV) to identify the causal effect of an endogenous treatment variable on an outcome of interest. For an IV to be valid, it must be an exogenous variable that only influences the outcome by affecting the treatment choice. However, this property alone does not ensure the nonparametric identification of treatment effects; additional assumptions are necessary to identify causal parameters. The general approach to secure identification is to impose choice restrictions that constraint how agents select the treatments as the instrument varies.

A seminal assumption for identifying treatment effects in binary choice models is the monotonicity condition of Imbens and Angrist (1994). The condition asserts that a change in the instrument can only induce agents to alter their choice in the same direction. Vytlacil (2002) demonstrates that this monotonicity condition is equivalent to assuming a separability condition in which the treatment choice is modeled by a threshold indicator function comparing the propensity score with a latent variable responsible for selection bias (Heckman and Vytlacil, 1999, 2005). These influential ideas spiked a vast literature on both empirical and theoretical aspects of identification assumptions in IV models with binary choices.[1]

The IV literature has successfully extended the notions of monotonicity and separability conditions to the case of multiple-choice models. The seminal work of Angrist and Imbens (1995) extends the previous monotonicity condition of Imbens and Angrist (1994) from the binary choice to the case of the multiple-choice model. Their condition applies to cases where the treatment variable has a natural order. Ordered choice models are also examined by Cameron and Heckman (1998) and further studied by Carneiro et al. (2003); Cunha et al. (2007).

A significant advance in the IV literature is due to Lee and Salanié (2018), who develop general identification results for multiple choice models characterized by a coherent set of separability conditions. On the realm of monotonicity conditions, Heckman and Pinto (2018) propose the unordered monotonicity condition that applies to treatment choices that are not ordered.[2] More recently, Rose and Shem-Tov (2021) proposes a monotonicity condition called extensive margin compliers only (EMCO), in which a change in the instrument incentives all agents to shift from no treatment to some treatment status. Mogstad et al. (2021a,b) investigate the monotonicity criteria in a choice model with multiple instrumental variables.

We propose a departure from the conventional mindset that guides the examination of identification assumptions in IV models. Instead of concentrating on devising new monotonicity or

[1]For examples of works in this literature, see Aliprantis (2012); Angrist et al. (2000); Barua and Lang (2016); Dahl et al. (2017); de Chaisemartin (2017); Heckman (2010); Heckman and Urzúa (2010); Heckman and Vytlacil (2007a,b); Huber et al. (2017); Huber and Mellace (2012, 2015); Hull (2018); Imbens and Rubin (1997); Klein (2010); Mogstad et al. (2018); Mogstad and Torgovitsky (2018); Small and Tan (2007).

[2]Unordered choice models have been studied mainly through the literature on structural equations. A common approach assumes that additively separable threshold-crossing models generate the equations that govern the treatment. Examples of this literature are Heckman et al. (2006, 2008); Heckman and Vytlacil (2007a,b).

separability conditions, we explore how economic incentives and classical economic behavior can be used to identify causal parameters in IV models featuring multiple choices and categorical instruments. Our method is rooted in a simple yet general framework that employs revealed preference analysis to transform choice incentives into identification conditions. We demonstrate that distinct patterns of choice incentives can offer economic justification for the identification assumptions frequently invoked in the IV literature. Furthermore, we demonstrate that the framework is a valuable tool for generating novel identification strategies that are economically justified.

Our framework offers a distinct advantage: identification relies not on statistical or functional form assumptions, but instead, identification conditions emerge from the application of fundamental principles of economic behavior to choice incentives. This feature enhances the credibility and comprehension of the identification mechanism. The method is flexible enough to accommodate a wide range of non-trivial identification assumptions. We demonstrate its flexibility by examining well-known examples of choice incentives in the literature on policy evaluation. The framework is also capable to provide innovative solutions to non-standard economic scenarios where the identification assumptions typically invoked by the IV literature do not apply.

We use our framework to investigate the migration of poor Mexican households to the US. Currently, the US houses approximately 12 million undocumented residents, with almost half originating from Mexico. Seminal work of Borjas (1987, 1994) suggests a negative selection in migration patterns, with lower-skilled workers benefiting the most from moving to the US. This perspective is supported by Angelucci (2015); Lange (2011), who show that schooling incentives led lower-skilled undocumented migrants to emigrate to the US. She uses data from Oportunidades, Mexico's flagship anti-poverty program (Gertler, 2004). On the other hand, Behrman et al. (2005); Chiquiar and Hanson (2005); Hanson (2006) posits a non-monotonic relationship between education and migration. Fundamental skills such as basic English proficiency taught in middle school increase the propensity to migrate. However, additional education reduces migration, since it makes the domestic labor market more attractive than its international counterpart.

We devise a stylized model that uses data from Oportunidades to assess the question of whether schooling has a non-monotonic effect on the decision to migrate to the US. We use our incentive framework to identify the causal effects of schooling on migration. We find compelling evidence that completing middle school increases the likelihood of migration, while advancing schooling beyond middle school does not. We estimate our model using novel machine learning techniques developed by Navjeevan, Pinto, and Santos (2023).

This paper adds to the economic literature that uses revealed preference analysis to aid the identification of causal parameters in policy evaluations. This recent literature has seen the emergence of seminal works such as Kline and Tartari (2016); Kline and Walters (2016b). In recent years, there has also been a growing interest in applying revealed preference analysis to IV models. This interest is reflected in the increasing number of studies that have employed revealed preference analysis to investigate IV models. Examples of works in this literature include Pinto (2022), Feller et al. (2016), Kamat (2021), Mountjoy (2021), and Brinch et al. (2017). Our empirical analysis adds to a significant literature that evaluates the Oportunidades intervention. Our findings corroborate the hypothesis of a non-monotonic relationship between the migration of poor Mexicans to the US and their education levels. Finally, this paper adds to a growing number of works that apply novel machine learning techniques (Chernozhukov et al., 2022; Smucler et al., 2019) to evaluate data.

This paper is organized as following. Section 2 describes our notation and a general criteria for the identification of causal parameters in IV models. Section 3 presents the revealed preference framework and shows how it relates to several works in the literature. Section 4 investigates how patterns of choice incentives yield identification conditions in IV models. Section 5 presents our empirical application. Section 6 concludes.

2 Setup and Notation

Our IV model consists of three observed variables: a categorical instrument Z that takes N_Z values in $\mathcal{Z} = \{z_1, ..., z_{N_Z}\}$; a multiple treatment choice T that takes N_T values in $\mathcal{T} = \{t_1, ..., t_{N_T}\}$; and an outcome $Y \in \mathbb{R}$. Let $Y(z,t)$ be the counterfactual outcome Y when (Z,T) are fixed to $(z,t) \in \mathcal{Z} \times \mathcal{T}$, and $Y(t)$ be the counterfactual outcome when T is fixed to $t \in \mathcal{T}$.[3] Let $D_t = \mathbf{1}[T = t]; t \in \mathcal{T}$ and $D_z = \mathbf{1}[Z = z]; z \in \mathcal{Z}$ denote binary indicators for treatment choices and IV-values respectively. The *core properties* of IV model for all $(z,t) \in \mathcal{Z} \times \mathcal{T}$ are:

$$\textbf{Exclusion Restriction :} \quad Y(z,t) = Y(t). \tag{1}$$

$$\textbf{IV Exogeneity:} \quad Z \perp\!\!\!\perp (Y(t), T(z)). \tag{2}$$

$$\textbf{IV Relevance:} \quad E\left([D_{z_1}, ..., D_{z_{N_Z}}]'[D_{t_l}, ..., D_{t_{N_T}}]\right) \text{ has full rank.} \tag{3}$$

The exclusion restriction implies that Z affects Y only through T. The exogeneity assumption means that the instrument Z is as good as randomly assigned, and the IV relevance states that Z causes T. All variables belong to the probability space $(\mathcal{I}, \mathcal{F}, P)$ where $i \in \mathcal{I}$ denotes an individual. We suppress baseline variables $\boldsymbol{X}$ for notational simplicity. All analyses can be understood as conditioned on $\boldsymbol{X}$.

The *response vector* $\boldsymbol{S} \equiv [T(z_1), ..., T(z_{N_Z})]'$ is the unobserved N_Z-dimensional vector that stacks the counterfactual choices $T(z)$ across the IV-values z in $\mathcal{Z}$. Elements $\boldsymbol{s}$ in the support of the response vector, $\mathcal{S} = \{\boldsymbol{s}_1, ..., \boldsymbol{s}_{N_S}\}$, are called *response types* or simply *types*. The *response matrix* $\boldsymbol{R} \equiv [\boldsymbol{s}_1, ..., \boldsymbol{s}_{N_S}]$ is the $N_Z \times N_S$ matrix that assembles types as columns. To fix ideas, consider the Local Average Treatment Effects (LATE) model of Imbens and Angrist (1994) with a binary instrument $Z \in \{z_0, z_1\}$ and a binary treatment $T \in \{t_0, t_1\}$. The response vector $\boldsymbol{S} = [T(z_0), T(z_1)]'$ admits four possible types: never-takers $\boldsymbol{s}_{\text{nt}} = [t_0, t_0]'$, compliers $\boldsymbol{s}_{\text{c}} = [t_0, t_1]'$, always-takers $\boldsymbol{s}_{\text{at}} = [t_1, t_1]'$, and defiers $\boldsymbol{s}_{\text{d}} = [t_1, t_0]'$. Without any additional assumption, its response matrix given by:

[3] This notation uses the potential outcome framework of Holland (1986); Rubin (1978). For a discussion on causality and the fixing operation, see Heckman and Pinto (2013, 2022).

$$
\boldsymbol{R} = \begin{array}{c} \\ \\ \end{array}\begin{array}{cccc} \boldsymbol{s}_{nt} & \boldsymbol{s}_{c} & \boldsymbol{s}_{at} & \boldsymbol{s}_{d} \end{array}
$$

$$
\boldsymbol{R} = \begin{array}{c} \begin{array}{cccc} \boldsymbol{s}_{nt} & \boldsymbol{s}_{c} & \boldsymbol{s}_{at} & \boldsymbol{s}_{d} \end{array} \\ \begin{bmatrix} t_0 & t_0 & t_1 & t_1 \\ t_0 & t_1 & t_1 & t_0 \end{bmatrix} \end{array} \begin{array}{l} \\ T(z_0) \\ T(z_1) \end{array} \tag{4}
$$

The response vector enables us to connect observed quantities with the unobserved counterfactuals according to the following equation:[4]

$$
\underbrace{E(Y|T=t,Z=z)P(T=t|Z=z)}_{\text{Observed}} = \sum_{\boldsymbol{s}\in\mathcal{S}} \underbrace{\mathbf{1}[T=t|\boldsymbol{S}=\boldsymbol{s},Z=z]}_{\text{Known}} \cdot \underbrace{E(Y(t)|\boldsymbol{S}=\boldsymbol{s})P(\boldsymbol{S}=\boldsymbol{s})}_{\text{Unobserved}} \ \forall (z,t)\in\mathcal{Z}\times\mathcal{T}. \tag{5}
$$

The left-hand side of equation (5) comprises of the observed quantities: the conditional expectation $E(Y|T=t,Z=z)$, and propensity score $P(T=t|Z=z)$.[5] The first term of the right-hand side of the equation is nonrandom since choice T is a fully determined given IV-value z and type $\boldsymbol{s}$. The second term on the right-hand comprises two unobserved quantities: the expected value of counterfactual outcomes conditioned on response types $E(Y(t)|\boldsymbol{S}=\boldsymbol{s})$, and the type probabilities $P(\boldsymbol{S}=\boldsymbol{s})$. It is useful to express equation (5) using the following matrix representation:

$$
\boldsymbol{Q}_Z(t)\odot\boldsymbol{P}_Z(t) = \boldsymbol{B}_t\left(\boldsymbol{Q}_S(t)\odot\boldsymbol{P}_S\right) \ \text{ for all } t\in\mathcal{T}, \tag{6}
$$

where $\boldsymbol{Q}_Z(t)\equiv[E(Y|T=t,Z=z_1),...,E(Y|T=t,Z=z_{N_Z})]'$ is the observed vector of outcome expectations; $\boldsymbol{P}_Z(t)\equiv[P(T=t|Z=z_1),...,P(T=t|Z=z_{N_Z})]$ is the observed vector of propensity scores; $\boldsymbol{Q}_S(t)\equiv[E(Y(t)|\boldsymbol{S}=\boldsymbol{s}_1),...,E(Y(t)|\boldsymbol{S}=\boldsymbol{s}_{N_S})]$, is the unobserved vector of counterfactual outcomes; $\boldsymbol{P}_S\equiv[P(\boldsymbol{S}=\boldsymbol{s}_1),...,P(\boldsymbol{S}=\boldsymbol{s}_{N_S})]$ is the unobserved vector of type probabilities; and $\odot$ denotes element-wise (Hadamard) multiplication. Finally, $\boldsymbol{B}_t\equiv\mathbf{1}[\boldsymbol{R}=t]$ is the $N_Z\times N_S$ binary matrix that takes value one if the entry in $\boldsymbol{R}$ is t and zero otherwise. Matrices $\boldsymbol{B}_t$ usually have full row rank since the number of columns N_S far exceeds the number of rows N_Z. We also use $\boldsymbol{B}_t[\cdot,\boldsymbol{s}]$ and $\boldsymbol{B}_t[z,\cdot]$ for the $\boldsymbol{s}$-column and z-row of $\boldsymbol{B}_t$ respectively. Under this notation, we can state the following identification criteria:[6]

Theorem T.1. Let $\boldsymbol{R}$ denotes a response matrix for a choice model in which IV Assumptions (1)–(3) hold and let $\tilde{\mathcal{S}}\subset\mathcal{S}$ be a subset of response types. For any choice $t\in\mathcal{T}$, such that the binary matrix $\boldsymbol{B}_t=\mathbf{1}[\boldsymbol{R}=t]$ has full row rank, we have that:

$$
E(Y(t)|\boldsymbol{S}\in\tilde{\mathcal{S}}) \text{ is identified } \quad\Leftrightarrow\quad \frac{\left(\sum_{\boldsymbol{s}\in\tilde{\mathcal{S}}}\boldsymbol{B}_t[\cdot,\boldsymbol{s}]\right)'\left(\boldsymbol{B}_t\boldsymbol{B}_t'\right)^{-1}\left(\sum_{\boldsymbol{s}\in\tilde{\mathcal{S}}}\boldsymbol{B}_t[\cdot,\boldsymbol{s}]\right)}{|\tilde{\mathcal{S}}|}=1,
$$

where $|\tilde{\mathcal{S}}|$ is the number of response types in the set $\tilde{\mathcal{S}}$. Moreover, if $E(Y(t)|\boldsymbol{S}\in\tilde{\mathcal{S}})$ is identified, it

[4]See Heckman and Pinto (2018) for the derivation of this equation.

[5]Equation (5) holds for any real-valued function $g:\mathbb{R}\to\mathbb{R}$ and for $(z,t)\in\mathcal{Z}\times\mathcal{T}$, that is:

$$
E(g(Y)|T=t,Z=z)P(T=t|Z=z)=\sum_{\boldsymbol{s}\in\mathcal{S}}\mathbf{1}[T=t|\boldsymbol{S}=\boldsymbol{s},Z=z]\cdot E(g(Y(t))|\boldsymbol{S}=\boldsymbol{s})P(\boldsymbol{S}=\boldsymbol{s}).
$$

Setting $g(Y)=\mathbf{1}[Y\leq y]; y\in\mathbb{R}$ generates an equation for the cumulative distribution function of counterfactual outcomes. Setting $g(Y)=1$ generates an equation that relates propensity scores and response type probabilities.

[6]The theorem also holds for the outcome transformation $g(Y)$ for any function $g:\mathbb{R}\to\mathbb{R}$.

can be evaluated by:

$$P(\boldsymbol{S} \in \tilde{\mathcal{S}}) = \Big(\sum_{\boldsymbol{s} \in \tilde{\mathcal{S}}} \boldsymbol{B}_t[\cdot, \boldsymbol{s}]\Big)' \left(\boldsymbol{B}_t \boldsymbol{B}_t'\right)^{-1} \boldsymbol{P}_Z(t),$$

$$E(Y(t)|\boldsymbol{S} \in \tilde{\mathcal{S}})P(\boldsymbol{S} \in \tilde{\mathcal{S}}) = \Big(\sum_{\boldsymbol{s} \in \tilde{\mathcal{S}}} \boldsymbol{B}_t[\cdot, \boldsymbol{s}]\Big)' \left(\boldsymbol{B}_t \boldsymbol{B}_t'\right)^{-1} \left(\boldsymbol{Q}_Z(t) \odot \boldsymbol{P}_Z(t)\right).$$

Proof. See Appendix A.1 □

The theorem presents a simple criterion for determining whether a counterfactual outcome $Y(t)$ is identified for a given set of types $\tilde{\mathcal{S}}$. In particular, it implies that for any choice t and any response type $\boldsymbol{s}$, the conditional expectation $E(Y(t)|\boldsymbol{S} = \boldsymbol{s})$ is identified if and only if $\boldsymbol{B}_t[\cdot, \boldsymbol{s}]' \left(\boldsymbol{B}_t \boldsymbol{B}_t'\right)^{-1} \boldsymbol{B}_t[\cdot, \boldsymbol{s}] = 1$.[7] A central message of this criterion is that the identification of the IV model relies solely on the properties of the response matrix $\boldsymbol{R}$. In other words, it stems from the selection of admissible types that constitute the response matrix $\boldsymbol{R}$. The next section proposes a framework that uses choice incentives to determine the set of admissible types in IV models.

3 Exploring Choice Incentives

Equation (5) establishes a system of linear equations in which the right-hand side comprises $N_Z \cdot N_T$ observed quantities, while the left-hand side consists of unobserved quantities that depend on the number of response types N_S. This system presents a fundamental identification problem. Without additional assumptions, the total number of types amounts to $N_T^{N_Z}$, which is significantly larger than the number of observed quantities $N_Z \cdot N_T$. This discrepancy precludes the point-identification of the unobserved quantities. Consequently, the identification of causal parameters hinges on assumptions that limit the number of admissible types N_S.

A well-known identification assumption in the LATE model of the previous section is the monotonicity condition of Imbens and Angrist (1994), which states that a change in the instrument from z_0 to z_1 induces agents to choose t_1. The condition is formalized as:

$$\mathbf{1}[T_i(z_0) = t_1] \leq \mathbf{1}[T_i(z_1) = t_1] \; \forall i. \tag{7}$$

The monotonicity condition eliminates the defiers ($\boldsymbol{s}_d$), which secures the identification of the causal effect for the compliers $E(Y(t_1) - Y(t_0)|\boldsymbol{S} = \boldsymbol{s}_c)$.

Extending the monotonicity condition of Imbens and Angrist (1994) to multiple choice models poses significant challenges. While the binary treatment model yields a single and intuitive monotonicity condition, the case of multiple choices presents a range of possibilities.[8] For instance, the monotonicity proposed by Angrist and Imbens (1995) and the unordered monotonicity of Heckman and Pinto (2018) differ significantly in multi-choice settings. However, both approaches collapse to the same monotonicity condition of Imbens and Angrist (1994) when the treatment is binary.

[7] Appendix A.2 demonstrates this result by applying the theorem to the LATE model.

[8] See Lee and Salanié (2018) for examples.

We argue that examining choice incentives offers a robust framework for investigating, generating, and justifying monotonicity conditions in multiple choice models. Some notation is required to translate these choice incentives into identification assumptions.

Let the *incentive matrix* $\boldsymbol{L}$ be a $N_Z \times N_T$-dimensional matrix that characterizes the choice incentives induced by the instrument. Each input $\boldsymbol{L}[z,t]$ denotes the relative incentive of the IV-value z (row) towards choice $t \in \mathcal{T}$ (column). The inequality $\boldsymbol{L}[z,t] \leq \boldsymbol{L}[z',t]$ means that a change in the IV from z to z' increases the incentives towards choice t, making the choice more attractive. In the case LATE, the IV value z_1 incentivizes choice t_1, while z_0 plays the role of a baseline comparison that does not incentivize any of the choices. Thus, LATE incentives can be characterized by the following matrix:

$$\text{LATE Incentive Matrix} \quad \boldsymbol{L} = \begin{array}{c} \begin{array}{cc} t_0 & t_1 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_0 \\ z_1 \end{array} \tag{8}$$

The first row of the matrix takes values zero since z_0 offers no incentives while the second row indicates that z_1 incentivizes t_1.

The choice rule below translates the choice incentives encoded by an incentive matrix $\boldsymbol{L}$ into choice restrictions that, in turn, eliminate response types:

$$\textbf{Choice Rule:} \quad \text{If } T_i(z) = t \text{ and } \boldsymbol{L}[z',t'] - \boldsymbol{L}[z,t'] \leq \boldsymbol{L}[z',t] - \boldsymbol{L}[z,t] \text{ then } T_i(z') \neq t'. \tag{9}$$

The choice rule esteems from standard revealed preference analysis.[9] It states that if agent i prefers choice t over t' under z-incentives ($T_i(z) = t$), and if z'-incentives favor t at least as much as t' ($\boldsymbol{L}[z',t'] - \boldsymbol{L}[z,t'] \leq \boldsymbol{L}[z',t] - \boldsymbol{L}[z,t]$), then the agent will not choose t' over t under z' ($T_i(z') \neq t'$). This rule highlights a fundamental principle of rational choice theory: an individual's preferences towards a choice remain consistent unless there are compelling incentives to choose otherwise. We demonstrate its application by revisiting key examples from the IV literature.

Example **E.1**. Our methodology offers an economic justification to the monotonicity condition in the LATE model. The condition emerges by applying revealed preference analysis captured by the choice rule (9) to the choice incentives represented by the incentive matrix (8):

$$T_i(z_0) = t_1, \text{ and } \boldsymbol{L}[z_1,t_0] - \boldsymbol{L}[z_0,t_0] = 0 \leq 1 = \boldsymbol{L}[z_1,t_1] - \boldsymbol{L}[z_0,t_1] \text{ thus } T_i(z_1) \neq t_0.$$

In summary, the LATE incentives lead to the choice restriction $T_i(z_0) = t_1 \Rightarrow T_i(z_1) \neq t_0$. It states that if agent i chooses t_1 under no incentives (z_0), it will not choose t_0 when the incentives for t_1 are present (z_1). This restriction eliminates the defiers and is equivalent to assume the customary monotonicity condition in (7). □

Next example investigates a multiple choice model.

Example **E.2**. Kline and Walters (2016a) evaluate the Head Start Impact Study using a model

[9] See Appendix A.3 for a formal derivation of the choice rule using revealed preference arguments.

with three treatment choices $T \in \{n, c, h\}$ corresponding to three day-care options: h stands for Head Start, c for other preschool programs, and n for no preschool (home-care). There are two instrumental values $Z \in \{z_0, z_1\}$, with z_1 indicating an offer to attend a Head Start school and z_0 if no offer is granted. The authors assume that the offer to attend Head Start offer does not induce children to leave Head Start nor instigate the children to switch between c and n. They express these assumptions by the following choice restriction:

$$T_i(z_0) \neq T_i(z_1) \Rightarrow T_i(z_1) = h \,\forall\, i \in \mathcal{I}. \tag{10}$$

This restriction can also be obtained by applying the choice rule to the model incentives. Specifically, the incentive matrix of the choice model is:

$$\boldsymbol{L} = \begin{array}{c} \\ \\ \end{array} \begin{array}{ccc} n & c & h \\ \end{array}$$
$$\boldsymbol{L} = \begin{array}{cc} \begin{array}{ccc} n & c & h \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} z_0 \\ z_1 \end{array} \end{array} \tag{11}$$

We can apply the choice rule in four instances:

$$\begin{aligned}
T_i(z_0) = h, \;\text{ and }\; \boldsymbol{L}[z_1, c] - \boldsymbol{L}[z_0, c] = 0 \leq 1 = \boldsymbol{L}[z_1, h] - \boldsymbol{L}[z_0, h] \;\text{ thus }\; T_i(z_1) \neq c, \\
T_i(z_0) = h, \;\text{ and }\; \boldsymbol{L}[z_1, n] - \boldsymbol{L}[z_0, n] = 0 \leq 1 = \boldsymbol{L}[z_1, h] - \boldsymbol{L}[z_0, h] \;\text{ thus }\; T_i(z_1) \neq n, \\
T_i(z_0) = n, \;\text{ and }\; \boldsymbol{L}[z_1, c] - \boldsymbol{L}[z_0, c] = 0 \leq 0 = \boldsymbol{L}[z_1, n] - \boldsymbol{L}[z_0, n] \;\text{ thus }\; T_i(z_1) \neq c, \\
T_i(z_0) = c, \;\text{ and }\; \boldsymbol{L}[z_1, n] - \boldsymbol{L}[z_0, n] = 0 \leq 0 = \boldsymbol{L}[z_1, c] - \boldsymbol{L}[z_0, c] \;\text{ thus }\; T_i(z_1) \neq n.
\end{aligned}$$

The first and second choice restrictions are summarized by $T_i(z_0) = h \Rightarrow T_i(z_1) = h$. The third restriction is $T_i(z_0) = n \Rightarrow T_i(z_1) \neq c$, and the fourth is $T_i(z_0) = c \Rightarrow T_i(z_1) \neq n$. Altogether, these restrictions are equivalent to the author's restriction in (10). These restrictions eliminate four of the nine possible types. The five response types that survive the elimination process are displayed in the following response matrix:

$$\boldsymbol{R} = \begin{array}{cc} \begin{array}{ccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 \end{array} & \\ \begin{bmatrix} n & c & h & n & c \\ n & c & h & h & h \end{bmatrix} & \begin{array}{c} T(z_0) \\ T(z_1) \end{array} \end{array} \qquad \square$$

A potential criticism of the revealed preference approach described here lies in its need for additional analytical tools, which may be deemed superfluous for examining simpler models like LATE. For these models, resorting solely to the monotonicity condition often presents a more straightforward approach. However, the advantages of the revealed preference framework become evident in the context of more intricate models, since the framework often outperforms the analyses that rely exclusively on monotonicity conditions. The following examples illustrate such cases.

Example **E.3**. Kirkeboen, Leuven, and Mogstad (2016) investigate a choice model featuring three treatment options (t_0, t_1, t_2) and three IV-values (z_0, z_1, z_2). In this model, z_1 incentivizes choice t_1, z_2 incentivizes t_2, and z_0 provides no incentives. The response vector is denoted by $\boldsymbol{S} = [T(z_0), T(z_1), T(z_2)]'$. There are a total of 27 potential response types since each of the three counterfactual choices $(T(z_0), T(z_1), T(z_2))$ can take on any of the three treatment values (t_0, t_1, t_2).

The choice incentives justify two monotonicity conditions:

$$\mathbf{1}[T_i(z_0)=t_1] \leq \mathbf{1}[T_i(z_1)=t_1], \quad \text{and} \quad \mathbf{1}[T_i(z_0)=t_2] \leq \mathbf{1}[T_i(z_2)=t_2]. \tag{12}$$

The first condition states that an IV-change from z_0 to z_1 induces agents to shift their choice toward t_1 while the second condition states that a change from z_0 to z_2 induces agents toward t_2. These monotonicity conditions eliminate 12 response types,[10] which is not sufficient to ensure the point-identification of any counterfactual outcome. The revealed preference approach delivers more identification power. The corresponding incentive matrix and the choice restrictions generated by the choice rule (9) are displayed below:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} t_0 & t_1 & t_2 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_0 \\ z_1 \\ z_2 \end{array} \quad \therefore \quad \begin{array}{lcl} T_i(z_0)=t_0 & \Rightarrow & T_i(z_1)\neq t_2 \text{ and } T_i(z_2)\neq t_1 \\ T_i(z_0)=t_1 & \Rightarrow & T_i(z_1)= t_1 \text{ and } T_i(z_2)\neq t_0 \\ T_i(z_0)=t_2 & \Rightarrow & T_i(z_1)\neq t_0 \text{ and } T_i(z_2)= t_2 \\ T_i(z_1)=t_2 & \Rightarrow & T_i(z_0)= t_2 \text{ and } T_i(z_2)= t_2 \\ T_i(z_2)=t_1 & \Rightarrow & T_i(z_0)= t_1 \text{ and } T_i(z_1)= t_1 \end{array} \tag{13}$$

The incentive matrix $\boldsymbol{L}$ denotes that z_1 incentivizes t_1 while the last row means that z_2 incentivizes t_2. Appendix A.4 applies choice rule (9) to each combination of two treatment values $(t,t') \in \{t_0,t_1,t_2\}$ and two instrumental values $(z,z') \in \{z_0,z_1,z_2\}^2$, which results in the five choice restrictions displayed above. The restrictions are intuitive. The first choice restriction states that if an agent chooses t_0 under z_0 (no incentives), then it will not choose t_2 under z_1, since z_1 does not incentivize t_2. The agent will not choose t_1 under z_2 either since z_2 does not incentivize t_1 either. In total, the five choice restrictions eliminate 19 response types, including the 12 types eliminated by the monotonicity conditions in (12).[11] The eight types that survive the elimination process are displayed in the following response matrix:

$$\boldsymbol{R} = \begin{array}{c} \begin{array}{cccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 \end{array} \\ \begin{bmatrix} t_0 & t_1 & t_2 & t_0 & t_0 & t_0 & t_1 & t_2 \\ t_0 & t_1 & t_2 & t_1 & t_0 & t_1 & t_1 & t_1 \\ t_0 & t_1 & t_2 & t_2 & t_2 & t_0 & t_2 & t_2 \end{bmatrix} \end{array} \begin{array}{c} \\ T(z_0) \\ T(z_1) \\ T(z_2) \end{array} \tag{14}$$

The first three response types, $\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{s}_3$, correspond to always-takers. They refer to agents that choose the same treatment choice (t_0, t_1, t_2 respectively) regardless of the instrumental value. Type $\boldsymbol{s}_4$ is called a full complier. It refers to agents that are most responsive to the IV incentives. They choose t_0 under no incentives, t_1 when assigned to z_1, and t_2 when assigned to z_2. The remaining four types $\boldsymbol{s}_5, ..., \boldsymbol{s}_8$ are called partial compliers since they choose two out of the three possible treatment statuses. □

Example **E.4**. Pinto (2022) examines the housing experiment called Moving to Opportunity. The model consists of three neighborhood choices t_h, t_m, t_l denoting high-, medium-, and low-poverty neighborhoods respectively. Families were randomly assigned to one of the three groups: the control group z_c offers no incentives, the Section Eight group z_8 received a housing voucher that incentivized

[10] The first monotonicity condition eliminates the six types given by $[t_1, t_2, t']$ or $[t_1, t_3, t']$ for $t' \in \{t_0, t_1, t_2\}$. The second monotonicity condition eliminates another six types: $[t_2, t', t_1]$ or $[t_2, t', t_3]$ for $t' \in \{t_0, t_1, t_2\}$. See Appendix A.4 for this analysis.

[11] See Appendix A.4 for the elimination process and additional analyses of this IV model.

families to choose either medium-poverty (t_m) or low-poverty (t_l) neighborhoods; the Experimental group z_e received a voucher that incentivized families to live in a low-poverty (t_l) neighborhoods; and the control group z_c received no voucher. These incentives justify three monotonicity conditions:

$$\mathbf{1}[T_i(z_c) = t_l] \leq \mathbf{1}[T_i(z_e) = t_l], \quad \mathbf{1}[T_i(z_c) \in \{t_m, t_l\}] \leq \mathbf{1}[T_i(z_8) \in \{t_m, t_l\}], \text{ and } \mathbf{1}[T_i(z_e) = t_m] \leq \mathbf{1}[T_i(z_8) = t_m].$$

The first conditions states that an IV change from z_c to z_e promotes t_h since z_c offers no incentives and z_e incentivizes only t_l. The second means that a change from z_c to z_8 incite choices t_m or t_h, since z_8 incentivizes both t_m and $_l$. The last condition means that a change from z_e to z_8 instigate choice t_m since both z_e, z_8 incentivize t_l but only z_8 incentivizes t_m. These conditions eliminate 14 out of the 27 response types which do not secure the point-identification of response type probabilities or counterfactual outcomes. The revealed preference analysis is more effective at eliminating response types. The incentive matrix and the corresponding choice restrictions are displayed below:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} t_h & t_m & t_l \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_c \\ z_8 \\ z_e \end{array} \quad \therefore \quad \begin{array}{lcl} T_i(z_c) = t_l & \Rightarrow & T_i(z_e) = t_l \text{ and } T_i(z_8) \neq t_h \\ T_i(z_c) = t_m & \Rightarrow & T_i(z_e) \neq t_h \text{ and } T_i(z_8) \neq t_h \\ T_i(z_e) = t_m & \Rightarrow & T_i(z_c) = t_m \text{ and } T_i(z_8) = t_m \\ T_i(z_e) = t_h & \Rightarrow & T_i(z_c) = t_h \text{ and } T_i(z_8) \neq t_l \\ T_i(z_8) = t_h & \Rightarrow & T_i(z_c) = t_h \text{ and } T_i(z_e) = t_h \\ T_i(z_8) = t_l & \Rightarrow & T_i(z_e) = t_l \\ T_i(z_c) \neq t_h & \Rightarrow & T_i(z_8) = T_i(z_c) \end{array} \tag{15}$$

These restrictions eliminate 20 out of the 27 possible response types including those eliminated by the monotonicity conditions. The seven types that survive the elimination process are displayed in the following response matrix:

$$\boldsymbol{R} = \begin{array}{c} \begin{array}{ccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 \end{array} \\ \begin{bmatrix} t_h & t_m & t_l & t_h & t_h & t_m & t_h \\ t_h & t_m & t_l & t_m & t_l & t_m & t_m \\ t_h & t_m & t_l & t_l & t_l & t_l & t_h \end{bmatrix} \end{array} \begin{array}{c} \\ T(z_c) \\ T(z_8) \\ T(z_e) \end{array} \tag{16}$$

These response types enable the point-identification of all response type probabilities and most of the counterfactual outcomes. □

Example **E.5**. Mountjoy (2022) studies the returns to two- and four-year college degrees. He uses proximity to college as an IV to encourage college enrollment. Let $T \in \{0, 2, 4\}$ represent the number of years of the college degree. The discrete version of the instrument is $Z = (Z_2, Z_4) \in \{0, 1\}^2$, where Z_2 and Z_4 indicate the proximity to two-year and four-year colleges, respectively. We use $T(z_2, z_4)$ for the counterfactual choice. The response vector $\boldsymbol{S} = [T(0,0), T(0,1), T(1,0), T(1,1)]'$ can take on the values of 81 potential response types. Proximity serves as an incentive for college enrollment, thereby justifying six natural monotonicity conditions:

$$\begin{aligned} &\mathbf{1}[T_i(1, z_4) = 0] \leq \mathbf{1}[T_i(0, z_4) = 0], && \mathbf{1}[T_i(z_2, 1) = 0] \leq \mathbf{1}[T_i(z_2, 0) = 0], \\ &\mathbf{1}[T_i(1, z_4) = 2] \geq \mathbf{1}[T_i(0, z_4) = 2], && \mathbf{1}[T_i(z_2, 1) = 2] \leq \mathbf{1}[T_i(z_2, 0) = 2], \\ &\mathbf{1}[T_i(1, z_4) = 4] \geq \mathbf{1}[T_i(0, z_4) = 4], && \mathbf{1}[T_i(z_2, 1) = 4] \geq \mathbf{1}[T_i(z_2, 0) = 4]. \end{aligned}$$

These monotonicity conditions state that an increase in the proximity to a two-year college induces agents towards choice 2 and away from choices 0 and 4. Conversely, an increase in the proximity to a four-year college induces agents towards choice 4 and away from choices 0 and 2. These monotonicity conditions eliminate 70 out of the 81 possible response types. The revealed preference analysis is capable of eliminating additional types. The incentive matrix of this choice model and its corresponding choice restrictions generated by the choice rule (9) are as follows:

$$
\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 0 & 2 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} (z_2, z_4) \\ (0,0) \\ (0,1) \\ (1,0) \\ (1,1) \end{array} \quad \therefore \quad
\begin{array}{lcl}
T_i(1,1)=0 & \Rightarrow & T_i(0,0)=0, \quad T_i(0,1)=0, \quad T_i(1,0)=0 \\
T_i(0,1)=0 & \Rightarrow & T_i(0,0)=0 \\
T_i(1,0)=0 & \Rightarrow & T_i(0,0)=0 \\
T_i(0,1)=2 & \Rightarrow & T_i(1,1)=2, \quad T_i(1,0)=2, \quad T_i(0,0)=2 \\
T_i(0,0)=2 & \Rightarrow & T_i(1,0)=2, \quad T_i(1,1)=2 \\
T_i(1,1)=2 & \Rightarrow & T_i(1,0)=2 \\
T_i(1,0)=4 & \Rightarrow & T_i(1,1)=4, \quad T_i(0,0)=4, \quad T_i(0,1)=4 \\
T_i(0,0)=4 & \Rightarrow & T_i(0,1)=4, \quad T_i(1,1)=4 \\
T_i(1,1)=4 & \Rightarrow & T_i(0,1)=4
\end{array} \tag{17}
$$

The incentive matrix generates nine choice restrictions that eliminate 72 response types, including those eliminated by the monotonicity conditions. The response matrix containing the nine types that survive this elimination process is displayed below:

$$
\boldsymbol{R} = \begin{array}{c} \begin{array}{ccccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 & \boldsymbol{s}_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2 & 2 & 4 & 4 \\ 0 & 0 & 4 & 4 & 4 & 2 & 4 & 4 & 4 \\ 0 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 4 \\ 0 & 2 & 4 & 2 & 4 & 2 & 2 & 4 & 4 \end{bmatrix} \end{array} \begin{array}{c} \\ T(0,0) \\ T(0,1) \\ T(1,0) \\ T(1,1) \end{array}
$$

□

Generality and Limitations

The framework presented here broadly applies to IV models that can be characterized by an incentive matrix. This scope encompasses instruments that span a wide range of incentives, which serve to augment the attractiveness, accessibility, or affordability of the treatment choices. Examples of such instruments include, but are not limited to, monetary rewards, advertising campaigns, tax reductions, subsidies, pricing mechanisms, and geographical proximity.

The incentive matrix exhibits several desirable properties: (1) The matrix allows an IV value to incentivize more than one treatment choice; (2) The generated choice restrictions are invariant to row and column permutations; (3) The restrictions are also invariant to any strictly monotonic transformations of the matrix; and (4) The restrictions are symmetric among choices that display the same incentives across IV-values.[12]

A key requirement of the incentive matrix is that treatment incentives must be comparable. For instance, the incentive matrix is not suitable to represent the choice incentives of a schooling experiment that seeks to boost academic performance by offering students monetary prizes or

[12] Identical rows of an incentive matrix mean that the corresponding IV-values are distinguishable in terms of choice incentives. In this case, it is advisable to merge these IV-values into a single representative value.

academic awards. These incentives are not easily ranked since some students may favor academic awards, while others gravitate towards monetary benefits.

4 What Incentives Justify Monotonicity Conditions?

Our framework provides a methodology for converting choice incentives into choice constraints. A natural application of this framework is to investigate which patterns of choice incentives justify monotonicity conditions typically invoked in the IV literature. Specifically, we study the economic content of Angrist and Imbens (1995) well-known monotonicity condition, the unordered monotonicity of Heckman and Pinto (2018), and the extensive margin compliers only (EMCO) discussed in Andresen and Huber (2021); Angrist and Imbens (1995); Rose and Shem-Tov (2021). We also study which incentive properties validate monotonicity conditions applicable to a single treatment choice in multiple-choice models.

All the theorems apply to the IV model described by Assumptions (1)–(3) whose choice incentives are given by an incentive matrix $\boldsymbol{L}$ that satisfies Choice Rule (9).

4.1 Investigating Ordered Monotonicity

Angrist and Imbens (1995) states that a change in the instrument induces all agents towards the same treatment direction:

$$T_i(z) \leq T_i(z') \,\forall\, i, \text{ or } T_i(z) \geq T_i(z') \,\forall\, i \text{ and any } z, z' \in \mathcal{Z}. \tag{18}$$

A celebrated result of Angrist and Imbens (1995) is that their monotonicity condition grants a causal interpretation to the standard Two-Stage Least Squares (2SLS) estimator. Vytlacil (2006) shows that this monotonicity condition is equivalent to assuming an ordered choice model with random thresholds. Thus, this monotonicity condition is commonly perceived as an intrinsic property of treatment choices that exhibit a natural order, such as years of schooling. This assessment is misleading. The primary feature of the condition is not the ordered nature of treatment choices. Instead, it is about a relationship between a sequence of IV-values whereby higher rankings of the z-values are associated with higher rankings of the counterfactual choices $T_i(z)$. To clarify, it is helpful to express this condition in terms of sequences of IV-values and treatment choices:

Ordered Monotonicity Condition (OMC): There exists an ordered sequence of treatment status $t_1 < \cdots < t_{N_T}$ in $\mathcal{T}$ and a sequence of IV-values $z_1, \ldots, z_{N_Z}$ in $\mathcal{Z}$ such that $T_i(z_1) \leq \cdots \leq T_i(z_{N_Z})$ holds for each $i \in \mathcal{I}$.

OMC is a slightly more inclusive version of the Angrist and Imbens (1995) condition. The monotonicity holds whenever it is possible to assign values to treatment choice T such that a sequence of IV-values produces an increasing sequence of counterfactual choices across all types. In the binary choice model, OMC is equivalent to the monotonicity condition of LATE. To gain intuition, consider the case where $T \in \{1, 2, 3\}$, $Z \in \{z_1, z_2, z_3\}$ and Ordered Monotonicity $T_i(z_1) \leq$

$T_i(z_2) \leq T_i(z_3)$ holds. The response matrix that contains all the admissible response types of model is:

$$\boldsymbol{R} = \begin{array}{c} \begin{array}{cccccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 & \boldsymbol{s}_9 & \boldsymbol{s}_{10} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 \\ 1 & 1 & 1 & 2 & 2 & 3 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 2 & 3 & 3 & 2 & 3 & 3 & 3 \end{bmatrix} \end{array} \begin{array}{l} \\ T(z_1) \\ T(z_2) \\ T(z_3) \end{array} \tag{19}$$

The matrix adheres to the OMC because choices weakly increase across successive rows. The response matrix contains ten types. In the general case of N_T choices and N_Z IV-values, the OMC yields a total of $\binom{N_T+N_Z-1}{N_T-1}$ admissible response types. A choice model is said to be *saturated* w.r.t. OMC if it contains every type that adheres to the condition. Otherwise stated, it is not possible to add another type without violating the condition. Conversely, a model is considered *unsaturated* w.r.t. OMC if yields only a subset of the possible types that adhere to the condition. This is the case of example **E.3**. Note that if we order the IV-values to (z_1, z_0, z_2) and assign the choice values $t_1 = 1, t_0 = 2, t_2 = 3$, we obtain the following response matrix representation:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ z_1 \\ z_0 \\ z_2 \end{array}, \quad \boldsymbol{R} = \begin{array}{c} \begin{array}{cccccccc} \boldsymbol{s}_2 & \boldsymbol{s}_1 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 \end{array} \\ \begin{bmatrix} 1 & 2 & 3 & 1 & 2 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 & 2 & 2 & 1 & 3 \\ 1 & 2 & 3 & 3 & 3 & 2 & 3 & 3 \end{bmatrix} \end{array} \begin{array}{l} \\ T(z_1) \\ T(z_0) \\ T(z_2) \end{array} \tag{20}$$

The treatment values of each type are non-decreasing as we move from one row to another. This pattern implies that $T_i(z_1) \leq T_i(z_0) \leq T_i(z_2)$ holds for all $i \in \mathcal{I}$ and therefore OMC is satisfied. The choice model is unsaturated w.r.t. OMC because it does not contain all the ten possible types. This means that the choice restrictions generated by the incentive matrix subsume and are more restrictive than the OMC.

We now examine the pattern of choice incentives that justifies the OMC. Incentives are termed *supermodular* if there exists a sequence of IV-values $z_1, ..., z_{N_Z}$ and a sequence of treatment choices $t_1, ...t_{N_T}$ such that, for any $j = 1, ..., N_Z - 1$, and $k = 1, ..., N_Z - 1$, we have that:

$$\textbf{Supermodular Incentives:} \ \boldsymbol{L}[z_{k+1}, t_j] - \boldsymbol{L}[z_k, t_j] \leq \boldsymbol{L}[z_{k+1}, t_{j+1}] - \boldsymbol{L}[z_k, t_{j+1}]. \tag{21}$$

Choice incentives are supermodular if the difference in incentives across IV-values weakly increases through the sequence of treatment choices. This pattern includes choice incentives that progressively escalate with higher ranks of IV-values and treatment statuses. Furthermore, we term an incentive matrix $\boldsymbol{L}$ *strictly supermodular* if the inequality in (21) is strictly enforced. We are now equipped to state the following result:

Theorem T.2. OMC holds if and only if incentives are supermodular. Moreover, a model with strictly supermodular incentives generates a saturated response matrix w.r.t. OMC.

Proof. See Appendix A.6. □

Theorem **T.2** states that supermodular incentives ensure the OMC. For notational convenience, let $\Delta\boldsymbol{L}$ be the row-difference of an incentive matrix $\boldsymbol{L}$:

$$\Delta\boldsymbol{L}[k,j] = (\boldsymbol{L}[z_{k+1},t_j] - \boldsymbol{L}[z_k,t_j]);\;\; k=1,..,N_Z-1;\; j=1,...,N_T.$$

Incentives are supermodular if $\Delta\boldsymbol{L}$ is weakly increasing in both row and column dimensions. In addition, the incentives are strictly supermodular the columns of $\Delta\boldsymbol{L}$ are strictly increasing, namely, $\Delta\boldsymbol{L}[\cdot,j] < \Delta\boldsymbol{L}[\cdot,j+1]$. Examples of supermodular incentives for $T \in \{1,2,3\}$ and $Z = \in \{z_1,z_2,z_3\}$ are:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \Rightarrow \Delta\boldsymbol{L} = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 1 & 5 \end{bmatrix} \tag{22}$$

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \Rightarrow \Delta\boldsymbol{L} = \begin{bmatrix} -1/2 & 0 & 1/2 \\ -1/2 & 0 & 1/2 \end{bmatrix} \tag{23}$$

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \Rightarrow \Delta\boldsymbol{L} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{24}$$

The first example (22) presents a Vandermonde matrix which exhibits increasing choice incentives in both row and the column dimensions. The matrix satisfies strictly supermodularity since the columns of $\Delta\boldsymbol{L}$ are strictly increasing. According to **T.2**, these incentives yield the saturated response matrix displayed in (19).

The second example (23) displays a choice pattern in which z_1 offers full incentives for choice 1; z_3 offers full incentives for choice 3; while z_2 splits the incentives between choices 1 and 3. Strictly supermodularity also holds since the columns of $\Delta\boldsymbol{L}$ are strictly increasing. Again, **T.2** ensures that these incentives also yield the saturated response matrix in (19).

The final incentive matrix (24) revisits the example in equation (20). This matrix exhibits supermodularity since the columns in $\Delta\boldsymbol{L}$ are weakly increasing. However, it does not fulfill the criteria for strict supermodularity because the columns of $\Delta\boldsymbol{L}$ are not strictly increasing. These incentives generate the unsaturated response matrix in (20), which satisfies OMC but has fewer types than the saturated version. It is worth noting that having fewer types confers additional identification power. It means that the incentive matrix yields choice restrictions that subsume and outperform the OMC.

4.2 Investigating Unordered Monotonicity

Heckman and Pinto (2018) propose an Unordered Monotonicity Condition that applies to treatment choices that are not ordered. The condition states that for each pair of IV-values $(z, z') \in \mathcal{Z}^2$ and for each $t \in \mathcal{T}$,

$$\mathbf{1}[T_i(z) = t] \leq \mathbf{1}[T_i(z') = t] \, \forall \, i \text{ or } \mathbf{1}[T_i(z) = t] \geq \mathbf{1}[T_i(z') = t] \, \forall \, i. \tag{25}$$

The condition means that for each of the choices t, an IV-change must induce induces all agents towards t or all agents away from t. Heckman and Pinto (2018) show that the condition naturally arises in a range of IV settings in which treatment choices do not have a clear ordering structure. They also show that unordered monotonicity enables us to express the indicator for choice t as a latent threshold indicator akin to the result in Vytlacil (2002). Specifically, $\mathbf{1}[T(z) = t] = \mathbf{1}[P_t(z) \geq U_t]$, where $P_t(z) = P(T = t|Z = z)$ is the propensity score and $U_t \sim Unif[0,1]$ is an unobserved random variable with uniform distribution in $[0,1]$ that is statistically independent of Z.[13] Pinto (2022) explores this choice representation to evaluate the Moving to Opportunity Intervention as described in example **E.4**. Finally, the monotonicity condition can be equivalently stated in terms of IV-sequences:

Unordered Monotonicity Condition (UMC): For each choice t, there exists a sequence of IV-values $(z_1^{(t)}, ..., z_{N_Z}^{(t)})$ in $\mathcal{Z}$ such that $\mathbf{1}[T_i(z_1^{(t)}) = t] \leq ... \leq \mathbf{1}[T_i(z_{N_Z}^{(t)}) = t]$.

UMC posits that for each treatment choice t there is a sequence of the IV-values that induce agents to choose t. These IV sequences can and do differ across choices $t \in \mathcal{T}$. In contrast, OMC employs a *single sequence* of IV values that induces all agents to choose higher treatment values.[14] In practical terms, UMC means that it is possible to reorder the rows and columns of the response matrix to generate a lower triangular matrix with respect to each choice t. We revisit the example **E.4** to illustrate this property:

$$\boldsymbol{R} = \begin{array}{c} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 \end{matrix} \\ \begin{bmatrix} t_h & t_m & t_l & t_h & t_h & t_m & t_h \\ t_h & t_m & t_l & t_m & t_l & t_m & t_m \\ t_h & t_m & t_l & t_l & t_l & t_l & t_h \end{bmatrix} \end{array} \begin{matrix} T(z_c) \\ T(z_8) \\ T(z_e) \end{matrix} \; , \quad \boldsymbol{R}_h = \begin{array}{c} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_7 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_6 \end{matrix} \\ \begin{bmatrix} t_h & t_m & t_m & t_l & t_m & t_l & t_m \\ t_h & t_h & t_l & t_l & t_m & t_l & t_l \\ t_h & t_h & t_h & t_h & t_m & t_l & t_m \end{bmatrix} \end{array} \begin{matrix} T(z_8) \\ T(z_e) \\ T(z_c) \end{matrix} \; , \tag{26}$$

$$\boldsymbol{R}_m = \begin{array}{c} \begin{matrix} \boldsymbol{s}_2 & \boldsymbol{s}_6 & \boldsymbol{s}_4 & \boldsymbol{s}_7 & \boldsymbol{s}_1 & \boldsymbol{s}_3 & \boldsymbol{s}_5 \end{matrix} \\ \begin{bmatrix} t_m & t_l & t_l & t_h & t_h & t_l & t_l \\ t_m & t_m & t_h & t_h & t_h & t_l & t_h \\ t_m & t_m & t_m & t_m & t_h & t_l & t_l \end{bmatrix} \end{array} \begin{matrix} T(z_e) \\ T(z_c) \\ T(z_8) \end{matrix} \; , \quad \boldsymbol{R}_l = \begin{array}{c} \begin{matrix} \boldsymbol{s}_3 & \boldsymbol{s}_5 & \boldsymbol{s}_4 & \boldsymbol{s}_6 & \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_7 \end{matrix} \\ \begin{bmatrix} t_l & t_h & t_h & t_m & t_h & t_m & t_h \\ t_l & t_l & t_m & t_m & t_h & t_m & t_m \\ t_l & t_l & t_l & t_l & t_h & t_m & t_h \end{bmatrix} \end{array} \begin{matrix} T(z_c) \\ T(z_8) \\ T(z_e) \end{matrix} \; . \tag{27}$$

The matrix $\boldsymbol{R}$ is the original response matrix of example **E.4**. Matrix $\boldsymbol{R}_h$ rearranges the columns and rows of the original matrix to generate a lower triangular matrix w.r.t. t_h. This ordering reveals

[13] Heckman and Pinto (2018) assume a general model where choice $T = f(Z, \boldsymbol{V})$ is a function of the instrument Z and an absolutely continuous unobserved random vector $\boldsymbol{V}$ that is statistically independent of Z.

[14] UMC does not imply or is implied by OMC in multiple choice models, but they do collapse to the monotonicity condition of Imbens and Angrist (1994) in the case of a binary choice.

that the number of types taking value t_h increase as we move along the IV-sequence z_8, z_e, z_c.[15] This means that the IV-sequence z_8, z_e, z_c induce agents to choose t_h and the following inequality holds:

$$\mathbf{1}[T_i(z_e) = t_h] \leq \mathbf{1}[T_i(z_8) = t_h] \leq \mathbf{1}[T_i(z_c) = t_h] \, \forall \, i \in \mathcal{I}.$$

Matrices $\boldsymbol{R}_m$ and $\boldsymbol{R}_l$ show that it is also possible to generate lower triangular matrices w.r.t. t_m and t_l via row and column permutations. Consequently, UMC is satisfied. In contrast, OMC does not hold because it is not possible to assign values to treatment choices t_h, t_m, t_l that ensure increasing sequences of counterfactuals $T_i(z_c) \leq T_i(z_8) \leq T_i(z_e)$ across all types.[16] Finally, the response matrix is said to be *saturated* w.r.t. the UMC because it is not possible to add another response type without violating UMC.

A binary matrix that can be transformed into lower triangular matrix via row and column permutations are called *lonesum* (Ryser, 1957). Thus we can state that UMC holds of and only if when all the binary matrices $\boldsymbol{B}_t \equiv \mathbf{1}[\boldsymbol{R} = t]; t \in \mathcal{T}$ are lonesum. A simple criterion to verify if a response matrix satisfies the UMC is to check if the matrix does not contain a 2×2 submatrix in which the diagonal contains a choice t and the off-diagonal does not.[17] This prohibit pattern prevents us to transform the response matrix into a lower triangular matrix of t-values as illustrated in equations (26)–(27). For example, response matrix (20) satisfies OMC but it does not satisfy UMC since 2×2 submatrix formed by columns $\boldsymbol{s}_5, \boldsymbol{s}_6$, and rows z_1, z_3, has choice 2 in its diagonal but lacks choice 2 in its off-diagonal.

We introduce the concept of *Monotonic incentives*, which is pivotal in analyzing the economic incentives underlying the UMC in binary incentive matrices.

$$\textbf{Monotonic Incentives:} \quad \text{for any } z, z' \in \mathcal{Z}, \quad \boldsymbol{L}[z', t] \leq \boldsymbol{L}[z, t] \forall \, t \in \mathcal{T} \text{ or } \boldsymbol{L}[z', t] \geq \boldsymbol{L}[z, t] \forall \, t \in \mathcal{T}. \tag{28}$$

Monotonic incentives imply the existence of an IV-sequence in which the incentives in $\boldsymbol{L}$ weakly increase for all choices. In the case of 3×3 binary matrices, there are four non-equivalent matrices that display monotonic incentives:

$$\boldsymbol{L} = \begin{array}{c c} \begin{matrix} 1 & 2 & 3 \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} & \begin{matrix} z_1 \\ z_2 \\ z_3 \end{matrix} \end{array}, \quad \boldsymbol{L} = \begin{array}{c c} \begin{matrix} 1 & 2 & 3 \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} & \begin{matrix} z_1 \\ z_2 \\ z_3 \end{matrix} \end{array}, \quad \boldsymbol{L} = \begin{array}{c c} \begin{matrix} 1 & 2 & 3 \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} & \begin{matrix} z_1 \\ z_2 \\ z_3 \end{matrix} \end{array}, \quad \boldsymbol{L} = \begin{array}{c c} \begin{matrix} 1 & 2 & 3 \end{matrix} & \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} & \begin{matrix} z_1 \\ z_2 \\ z_3 \end{matrix} \end{array}.$$

These matrices satisfy the monotonic incentive criteria because $\boldsymbol{L}[z_1, t] \leq \boldsymbol{L}[z_2, t] \leq \boldsymbol{L}[z_3, t]$ holds for all $t \in \{t_1, t_2, t_3\}$. They are lonesum since they are binary and triangular. Indeed, if $\boldsymbol{L}$ is binary, the concept of monotonic incentives is equivalent to $\boldsymbol{L}$ being lonesum. In particular, the first incentive matrix above is equivalent to the one in the example E.4.[18] Next theorem explains

[15] Under z_8, only $\boldsymbol{s}_1$ takes the value t_h. Under z_e, the types $\boldsymbol{s}_1$ and $\boldsymbol{s}_7$ take the value t_h, and under z_c, the types that take the value t_h are $\boldsymbol{s}_1, \boldsymbol{s}_7, \boldsymbol{s}_4, \boldsymbol{s}_5$.

[16] For instance, selecting choice values such that $t_h < t_m < t_l$ results in an increasing sequence of treatment values for response type $\boldsymbol{s}_4 = [t_h, t_m, t_l]'$, but it fails to generate an increasing sequence for the type $\boldsymbol{s}_7 = [t_h, t_m, t_h]'$.

[17] See Heckman and Pinto (2018) for a discussion on these properties.

[18] Two matrices are said to be equivalent if they have the same dimension and one can be transformed into the other via row and column permutations.

that monotonic incentives imply UMC:

Theorem T.3. UMC holds for all binary incentive matrices $\boldsymbol{L}$ satisfying monotonic incentives.

Proof. See Appendix A.7. $\square$

An alternative way to express the Theorem **T.3** is:

Corollary C.1. $\boldsymbol{L}$ is lonesum $\Rightarrow$ $\boldsymbol{B}_t$ is lonesum for all $t \in \mathcal{T}$.

The corollary states that if the Incentive matrix $\boldsymbol{L}$ is lonesum,then it can be transformed into a lower triangular matrix via row and column permutations. This means that monotonic incentives hold. Applying the choice rule to the incentive matrix generates choice restrictions that yield a response matrix $\boldsymbol{R}$ that satisfy UMC, and thereby, the binary matrices $\boldsymbol{B}_t \equiv \mathbf{1}[\boldsymbol{R} = t]$ are lonesum, which means that these matrices can be transformed into lower triangular matrices via row and column permutations.

The criterion of monotonic incentives (28) is instrumental in ensuring that UMC holds. However, this criterion does not encompass all types of incentives capable of inducing UMC. To establish a more general criterion, it is necessary to study the incentive patterns that induce monotonicity for a single choice t, namely:

$$\mathbf{1}[T_i(z) = t] \leq \mathbf{1}[T_i(z') = t] \; \forall\, i \text{ or } \mathbf{1}[T_i(z) = t] \geq \mathbf{1}[T_i(z') = t] \; \forall\, i \in \mathcal{I} \text{ and any } z, z' \in \mathcal{Z}. \tag{29}$$

The monotonicity condition above describes an indicator inequality in which a change in the instrument induce all agents towards choice t or away from choice t. The condition focuses on a single choice t. UMC arises when this condition holds for all $t \in \mathcal{T}$. The t-monotonic incentives, described below, is central in generating the monotonicity condition of the choice indicator (29):

t-Monotonic Incentives: $\boldsymbol{L}$ is t-monotonic if, for any two IV-values $z, z' \in \mathcal{Z}$, we have that:

$$\boldsymbol{L}[z', t] - \boldsymbol{L}[z, t] \leq \boldsymbol{L}[z', t'] - \boldsymbol{L}[z, t'] \; \forall\, t' \in \mathcal{T} \text{ or } \boldsymbol{L}[z', t] - \boldsymbol{L}[z, t] \geq \boldsymbol{L}[z', t'] - \boldsymbol{L}[z, t'] \; \forall\, t' \in \mathcal{T}. \tag{30}$$

Incentives are termed t-monotonic if for any instrumental change the incentive difference for the choice t is either the maximum or the minimum incentive difference among all treatment choices. The next theorem describes the relevant properties of t-monotonic incentives:

Theorem T.4. Monotonicity condition (29) holds for choice t if and only if $\boldsymbol{L}$ is t-monotonic.

Proof. See Appendix A.8. $\square$

The theorem states that t-monotonic incentives is a necessary and sufficient condition for the indicator monotonicity condition (29) to hold. A natural consequence of the theorem is:

Corollary C.2. UMC holds if and only if $\boldsymbol{L}$ is t-monotonic for all $t \in \mathcal{T}$.

The corollary directly stems from Theorem **T.4** and the definition of UMC, which asserts that the indicator monotonicity condition (29) applies to all choices $t \in \mathcal{T}$. Essentially, the corollary states that given an IV-change, UMC holds only if the incentive differences for each of the choice $t \in \mathcal{T}$ is either the maximum or the minimum of the differences across all choices. This implies that the incentive differences should exhibit at most two distinct values throughout all treatment values. The following examples help clarify this property:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} t_h & t_m & t_l \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_c \\ z_8 \\ z_e \end{array} \therefore \begin{array}{c|ccc} & t_h & t_m & t_l \\ \hline \boldsymbol{L}[z_8,t]-\boldsymbol{L}[z_c,t] & 0 & 1 & 1 \\ \boldsymbol{L}[z_e,t]-\boldsymbol{L}[z_c,t] & 0 & 1 & 0 \\ \boldsymbol{L}[z_8,t]-\boldsymbol{L}[z_e,t] & 0 & 0 & 1 \end{array} \tag{31}$$

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \therefore \begin{array}{c|ccc} & t_1 & t_2 & t_3 \\ \hline \boldsymbol{L}[z_2,t]-\boldsymbol{L}[z_1,t] & 1 & 0 & 0 \\ \boldsymbol{L}[z_3,t]-\boldsymbol{L}[z_1,t] & 0 & 1 & 1 \\ \boldsymbol{L}[z_3,t]-\boldsymbol{L}[z_2,t] & -1 & 1 & 1 \end{array} \tag{32}$$

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \end{array} \therefore \begin{array}{c|ccc} & t_1 & t_2 & t_3 \\ \hline \boldsymbol{L}[z_2,t]-\boldsymbol{L}[z_1,t] & 0 & 1 & 1 \\ \boldsymbol{L}[z_3,t]-\boldsymbol{L}[z_1,t] & 1 & 2 & 2 \\ \boldsymbol{L}[z_3,t]-\boldsymbol{L}[z_2,t] & 1 & 1 & 1 \end{array} \tag{33}$$

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ z_1 \\ z_0 \\ z_2 \end{array} \therefore \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline \boldsymbol{L}[z_0,t]-\boldsymbol{L}[z_1,t] & -1 & 0 & 0 \\ \boldsymbol{L}[z_2,t]-\boldsymbol{L}[z_1,t] & -1 & 0 & 1 \\ \boldsymbol{L}[z_2,t]-\boldsymbol{L}[z_0,t] & 0 & 0 & -1 \end{array} \tag{34}$$

Equation (31) investigates the incentive matrix of example **E.4**. The second matrix displays the incentive differences corresponding to IV-changes (rows) across the treatment statuses (columns). These incentive differences take only two values, zero or one. Thus, for each IV-comparison, the incentive difference of any treatment status is either the maximum or the minimum among the possible values. This property imply that the incentive matrix is t-monotonic for each treatment choice and, according to **C.2**, UMC holds. This result was previously assessed by noting that the matrix is a case of monotonic incentives (28).

Equation (32) presents a binary incentive matrix that does not exhibit monotonic incentives since the IV-change from z_2 to z_3 decreases the incentive for choosing t_1 while increases the incentive for choosing t_2. However the incentive differences for each IV-change (row) take only two values across all treatment statuses. Thus, for every IV-comparison, the incentive difference of each treatment choice takes either the maximum or the minimum. Consequently, UMC also holds. Equation (33) presents an incentive matrix that is not binary. The incentive differences associated for each IV-change also take at most two values across the treatment choices, which imply UMC as well. See The corresponding response matrices and a more detailed analysis of these models can be found in Appendix A.9.

Equation(34) reexamines the incentive matrix in (20), where OMC holds. The incentive difference $\boldsymbol{L}[z_2,t]-\boldsymbol{L}[z_1,t]$ takes three values across $t \in \{1,2,3\}$. Thus, according to **C.2**, UMC does not hold. This fact is corroborated by examining the response matrix in (20). The matrix displays a prohibit pattern in the 2×2 submatrix composed of types $\boldsymbol{s}_5, \boldsymbol{s}_6$ and rows z_1, z_2. The submatrix contains the value two in its the diagonal, but does not contain two in its off-diagonal.

We can further explore the properties of t-monotonic incentives. A simple method to check for t-monotonicity in binary incentive matrices is to split the incentive matrix $\boldsymbol{L}$ into $\boldsymbol{L}_t^0$ and $\boldsymbol{L}_t^1$ such that $\boldsymbol{L}_t^0$ contains the z-rows $\boldsymbol{L}[z,\cdot]$ such that $\boldsymbol{L}[z,t]=0$ and $\boldsymbol{L}_t^1$ contains the z-rows such that $\boldsymbol{L}[z,t]=1$. Under this notation, we can present the following result:

Corollary C.3. For any binary incentive matrix $\boldsymbol{L}$, incentives are t-monotonic for a choice $t \in \mathcal{T}$ if and only if matrices $\boldsymbol{L}_t^1$ and $\boldsymbol{L}_t^0$ are lonesum.

Proof. See Appendix A.10. $\square$

The corollary states that a necessary and sufficient condition for t-monotonicity to hold in a binary incentive matrix $\boldsymbol{L}$ is that matrices $\boldsymbol{L}_t^1$ and $\boldsymbol{L}_t^0$ are lonesum matrices. We use the incentive matrix of example **E.5** to illustrate an application of the corollary:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{cccc} 0 & 2 & 4 & (z_2,z_4) \end{array} \\ \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{c} (0,0) \\ (0,1) \\ (1,0) \\ (1,1) \end{array} \end{array} \therefore \quad \begin{array}{c} \boldsymbol{L}_2^0 = \begin{array}{c} \begin{array}{cccc} 0 & 2 & 4 & (z_2,z_4) \end{array} \\ \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right] \begin{array}{c} (0,0) \\ (0,1) \end{array} \end{array} \\ \\ \boldsymbol{L}_2^1 = \begin{array}{c} \begin{array}{cccc} 0 & 2 & 4 & (z_2,z_4) \end{array} \\ \left[\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{c} (1,0) \\ (1,1) \end{array} \end{array} \end{array} , \quad \begin{array}{c} \boldsymbol{L}_4^0 = \begin{array}{c} \begin{array}{cccc} 0 & 2 & 4 & (z_2,z_4) \end{array} \\ \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \begin{array}{c} (0,0) \\ (1,0) \end{array} \end{array} \\ \\ \boldsymbol{L}_4^1 = \begin{array}{c} \begin{array}{cccc} 0 & 2 & 4 & (z_2,z_4) \end{array} \\ \left[\begin{array}{ccc} 0 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{c} (0,1) \\ (1,1) \end{array} \end{array} \end{array} .$$

The first matrix displays the incentive matrix $\boldsymbol{L}$ of example **E.5**. Matrices $\boldsymbol{L}_2^0$ and $\boldsymbol{L}_2^1$ split $\boldsymbol{L}$ according to the incentives of choice 2. These matrices do not contain the prohibit pattern (the 2×2 identity matrix). Thus, according to **C.3**, the incentive matrix $\boldsymbol{L}$ is 2-monotonic. Matrices $\boldsymbol{L}_4^0$ and $\boldsymbol{L}_4^1$ refer to choice 4. These matrices do not present the prohibited pattern either. Therefore, $\boldsymbol{L}$ is also 4-monotonic. Incentives for choice 0, first column of $\boldsymbol{L}$, are all zero. Thus $\boldsymbol{L}_0^0 = \boldsymbol{L}$, and the matrix displays the prohibit pattern in the columns associated with choices 2 and 4, and rows $(0,1)$ and $(1,0)$. Therefore $\boldsymbol{L}$ is not 0-monotonic and UMC does not hold.

Although UMC does not hold, the incentive matrix is t-monotonic with respect to choices 2 and 4. Consequently, the monotonicity condition (30) holds for these two choices. This means that there must exist two IV-sequences capable of induce agents each of each of these choices. Indeed, we can reorder the rows and columns of the response matrix to review a progressive and monotonic pattern of choice selection:

$$\boldsymbol{R}_2 = \begin{array}{c} \begin{array}{ccccccccc} \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_2 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_8 & \boldsymbol{s}_1 & \boldsymbol{s}_3 & \boldsymbol{s}_9 \end{array} \\ \begin{bmatrix} 2 & 4 & 0 & 4 & 4 & 4 & 0 & 4 & 4 \\ 2 & 2 & 0 & 0 & 0 & 4 & 0 & 0 & 4 \\ 2 & 2 & 2 & 2 & 4 & 4 & 0 & 4 & 4 \\ 2 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 4 \end{bmatrix} \end{array} \begin{array}{l} \\ T(0,1) \\ T(0,0) \\ T(1,1) \\ T(1,0) \end{array}$$

$$\boldsymbol{R}_4 = \begin{array}{c} \begin{array}{ccccccccc} \boldsymbol{s}_9 & \boldsymbol{s}_8 & \boldsymbol{s}_3 & \boldsymbol{s}_5 & \boldsymbol{s}_4 & \boldsymbol{s}_7 & \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_6 \end{array} \\ \begin{bmatrix} 4 & 2 & 0 & 2 & 2 & 2 & 0 & 2 & 2 \\ 4 & 4 & 0 & 0 & 0 & 2 & 0 & 0 & 2 \\ 4 & 4 & 4 & 4 & 2 & 2 & 0 & 2 & 2 \\ 4 & 4 & 4 & 4 & 4 & 4 & 0 & 0 & 2 \end{bmatrix} \end{array} \begin{array}{l} \\ T(0,1) \\ T(0,0) \\ T(1,1) \\ T(0,1) \end{array}$$

The equations above show that the response matrix can be reordered into a lower triangular matrix with respect to choices 2 and 4. This corroborates the result that the monotonicity condition (30) holds for choices 2 and 4. The same feature does not apply to choice 0 since the 2×2 submatrix of response types $\boldsymbol{s}_2$ and $\boldsymbol{s}_3$ and rows $T(0,1)$ and $T(1,0)$ displays the prohibit pattern. As expected, UMC does not hold. OMC does not hold either since no IV-sequence yields a weakly increasing sequence of treatment choices across all types. Finally, the t-monotonicity conditions enables us to express the choice model by the following structural equations:

$$T = \begin{cases} 0 & \text{if } P_2(Z) < U_2 \text{ and } P_4(Z) < U_4, \\ 2 & \text{if } P_2(Z) \geq U_2, \\ 4 & \text{if } P_4(Z) \geq U_4, \end{cases}$$

where $P_t(Z) \equiv P(T = t|Z)$, $U_t \sim Unif[0,1]$, and $Z \perp\!\!\!\perp U_t$ for $t \in \{2,4\}$. This structural representation arises from the t-monotonicity of choices 2 and 4, and the fact that choice 0 is the complement of choices 2 and 4. This representation allows us to express the counterfactual outcomes $Y(2), Y(4)$ as functions of propensity scores $P_2(Z)$ and $P_4(Z)$ respectively, while $Y(0)$ is a function of both propensity scores. Additional identification power emerges when assuming functional forms for these counterfactuals or exploring baseline variables to generate variation in propensity scores. In the case of continuous instruments, this structural representation can be used to identify average treatment effects using the framework proposed by Lee and Salanié (2018).

It is helpful to summarize our analytical progress thus far. We have shown that the incentive matrix of example **E.3** generate a choice model satisfying only OMC. The choice incentives of example **E.4** generates a model adhering solely to UMC. Furthermore, the incentives of example **E.5** result in a choice model wherein either UMC or OMC is applicable. Next section explores incentives that lead to choice models where both UMC and OMC hold.

4.3 Incentives that Justify Recoding Treatment into an Exposure Indicator

The empirical analysis of IV models frequently involves the conversion of a multi-valued treatment into a binary variable that indicates exposure to a treatment. A typical example is to recode years of schooling into a dummy variable for college or high school graduation.[19] Angrist and Imbens (1995) argue that recoding the treatment status is problematic since the common 2SLS estimand recovers a weighted average of effects that usually do not have the intended causal interpretation. This problem has been recently studied by Andresen and Huber (2021) and Rose and Shem-Tov (2023). A simple example clarifies this issue.

Consider the IV model where $T \in \{0, 2, 4\}$ denotes years of college education. Let $Z \in \{z_0, z_1\}$, be an instrument where z_1 offers increasing incentives to greater years of college education while z_0 is a baseline comparison that offers no choice incentives:

$$\boldsymbol{L} = \begin{array}{c} \begin{array}{ccc} 0 & 2 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \\ z_0 \\ z_1 \end{array} \quad \therefore \quad \boldsymbol{R} = \begin{array}{c} \begin{array}{cccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 \end{array} \\ \begin{bmatrix} 0 & 2 & 4 & 0 & 0 & 2 \\ 0 & 2 & 4 & 2 & 4 & 4 \end{bmatrix} \end{array} \begin{array}{c} \\ T(z_0) \\ T(z_1) \end{array}$$

The incentive matrix above satisfies strict supermodularity, which yields a saturated response matrix with respect to OMC. The corresponding response matrix has six types $\boldsymbol{s}_1$–$\boldsymbol{s}_6$ ensuring that $T_i(z_0) \leq T_i(z_1)$ holds for all $i \in \mathcal{I}$. Types $\boldsymbol{s}_1$–$\boldsymbol{s}_3$ are always-takers, while $\boldsymbol{s}_4$–$\boldsymbol{s}_6$ are compliers. Suppose a research intends to evaluate the causal effect of four-year college graduation and thus recodes the treatment T into the binary variable $D = \mathbf{1}[T = 4]$ that indicates if the agent has completed a four-year college education. The Wald estimand of the 2SLS regression recovers the following causal response:

$$\frac{E(Y|Z=z_1) - E(Y|Z=z_0)}{E(D|Z=z_1) - E(D|Z=z_0)} = \underbrace{\frac{E(Y(4)-Y(0)|\boldsymbol{s}_5)P(\boldsymbol{s}_5) + E(Y(4)-Y(2)|\boldsymbol{s}_6)P(\boldsymbol{s}_6)}{P(\boldsymbol{S} \in \{\boldsymbol{s}_5, \boldsymbol{s}_6\})}}_{\text{Intended Effect (extra-margin)}} + \underbrace{\frac{E(Y(2)-Y(0)|\boldsymbol{s}_4)P(\boldsymbol{s}_4)}{P(\boldsymbol{S} \in \{\boldsymbol{s}_5, \boldsymbol{s}_6\})}}_{\text{Unintended Effect (intra-margin)}}$$

This estimand presents two problems. First, it conflates an intended effect with an unintended one. The *intended effect* is the weighted average of the causal effect of four-year college graduation against no college, $E(Y(4) - Y(0)|\boldsymbol{s}_5)$, and two-year college, $E(Y(4) - Y(2)|\boldsymbol{s}_6)$. These effects refer to types $\boldsymbol{s}_5$ and $\boldsymbol{s}_6$ which display *extra-margin variation*: T shifts from 0 or 2 to 4 when D changes from zero to one. On the other hand, he *unintended effect* evaluates the causal effect of two-year college versus no college, $E(Y(2) - Y(0)|\boldsymbol{s}_4)$. This effect refers to type $\boldsymbol{s}_4$ which displays an *intra-margin variation*: T changes from 0 to 2 while D remains constant. The second problem, highlighted by Andresen and Huber (2021), is that the binary treatment violates the IV exclusion restriction since the IV affects the counterfactual outcomes through channels beyond D. A solution to both problems is to prevent intra-margin treatment variation by eliminating type $\boldsymbol{s}_4$.

Rose and Shem-Tov (2021) coined the term Extensive Margin Compliers Only (EMCO) for a monotonicity condition that prevents intra-margin treatment variation in IV models with a binary instrument. We present a revised condition that extends their approach to the case of a categorical

[19] Numerous empirical studies undertake a binary conversion of a multi-valued treatment, including Aizer and Doyle (2015); Arteaga (2021); Bhuller et al. (2020); Black et al. (2005); Carneiro et al. (2011); Finkelstein et al. (2012); Kane and Rouse (1995); Mogstad and Wiswall (2016).

instrument. For a given treatment status $t \in \mathcal{T}$ and any for any $z, z' \in \text{supp}(Z)$ we have that:

$$t\textbf{-EMCO:} \quad \mathbf{1}[T_i(z) = t] \leq \mathbf{1}[T_i(z') = t] \forall\, i \text{ or } \mathbf{1}[T_i(z) = t] \geq \mathbf{1}[T_i(z') = t] \forall\, i \tag{35}$$

$$\text{and } T_i(z) \neq T_i(z') \Rightarrow T_i(z) = t \text{ or } T_i(z') = t. \tag{36}$$

The t-EMCO condition refers to a single treatment status t and combines two requirements: the monotonicity condition of the choice indicator (35), and a *no intra-margin condition* (36), which ensures that any choice shift within each complier must be confined between t and one other choice. The following response matrix displays the response types for $T \in \{0, 2, 4\}$ and $Z \in \{z_1, z_2, z_3\}$, when 4-EMCO holds:

$$\begin{array}{cc} & \begin{array}{ccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 \end{array} \\ \boldsymbol{R} = & \begin{bmatrix} 0 & 2 & \boxed{4} & 0 & 2 & 0 & 2 \\ 0 & 2 & \boxed{4} & \boxed{4} & \boxed{4} & 0 & 2 \\ 0 & 2 & \boxed{4} & \boxed{4} & \boxed{4} & \boxed{4} & \boxed{4} \end{bmatrix} \begin{array}{l} T(z_1) \\ T(z_2) \\ T(z_3) \end{array} \end{array} \tag{37}$$

Response types $\boldsymbol{s}_1 - \boldsymbol{s}_3$ are always-takes. The remaining types $\boldsymbol{s}_6 - \boldsymbol{s}_7$ are the compliers. The monotonicity of the choice indicator holds since $\mathbf{1}[T_i(z_1) = 4] \leq \mathbf{1}[T_i(z_2) = 4] \leq \mathbf{1}[T_i(z_3) = 4]$. Compliers do not display intra-margin variation. The counterfactual choices vary only between two treatment statuses: choices 4 or 0 in $\boldsymbol{s}_4$ and $\boldsymbol{s}_6$, and choices 4 or 2 in $\boldsymbol{s}_5$ and $\boldsymbol{s}_7$. This feature ensures that the Wald estimand evaluates a weighted average of intended treatment effects.

Theorem T.5. t-EMCO implies OMC and UMC.

Proof. See Appendix A.11. □

The t-EMCO is a particular case of monotonicity condition that satisfies both OMC and UMC. Indeed, a saturated response matrix with respect to EMCO is also saturated with respect to UMC, but unsaturated with respect to OMC. Results associated with OMC and UMC apply. In particular, the 2SLS estimand evaluates a weighted average of per-unit treatment effects that compares choice t with the remaining choices across compliers (Angrist and Imbens, 1995), and each choice can be expressed by a separable equation on the propensity score and a latent variable (Heckman and Pinto, 2018).

An incentive matrix $\boldsymbol{L}$ is said to satisfy the t-EMCO incentives when the following condition applies:

$$t\textbf{-EMCO Incentives}: \quad \boldsymbol{L}[z, t] = \boldsymbol{L}[z', t]\ \forall z, z' \in \text{supp}(Z), \tag{38}$$

$$\text{and } \boldsymbol{L}[z, t'] = \boldsymbol{L}[z, t'']\ \forall t', t'' \in \mathcal{T} \setminus \{t\} \text{ and } z \in \text{supp}(Z). \tag{39}$$

This condition states that incentives for choice t are constant *across* all IV values, while the incentives for the remaining choices are the same for any *given* IV-value. We can now state the following result:

Theorem T.6. If $\boldsymbol{L}$ satisfy the t-EMCO incentives (38), then t-EMCO (35) holds.

Proof. See Appendix A.12. ☐

Consider the IV example of this section where $T \in \{0, 2, 4\}$ and $Z \in \{z_1, z_2, z_3\}$. Examples of 4-EMCO incentives that generate the response matrix in (37) are:

$$\boldsymbol{L} = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cc} \begin{array}{ccc} 0 & 2 & 4 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 1/2 & 0 \\ 1 & 1 & 0 \end{bmatrix} & \begin{array}{c} z_3 \\ z_2 \\ z_1 \end{array} \end{array}, \quad \boldsymbol{L} = \begin{array}{cc} \begin{array}{ccc} 0 & 2 & 4 \end{array} & \\ \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix} & \begin{array}{c} z_3 \\ z_2 \\ z_1 \end{array} \end{array}, \quad \boldsymbol{L} = \begin{array}{cc} \begin{array}{ccc} 0 & 2 & 4 \end{array} & \\ \begin{bmatrix} -1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} & \begin{array}{c} z_3 \\ z_2 \\ z_1 \end{array} \end{array}$$

The key feature of these incentive matrices is that the incentives for choice 4 remain constant among IV values while the incentives for choices 2 and 4 are the same given each IV-value. It is easy to verify that EMCO incentives are monotonic and supermodular, which yield a response matrix that jointly satisfy OMC and UMC.

5 An Empirical Exercise

We use our incentives framework to examine how human capital impacts the illegal migration of poor Mexican families to the US. The US hosts approximately 12 million undocumented residents, with nearly half originating from Mexico. According to Borjas (1987), migration decisions are positively influenced by wage differentials between origin and destination countries but are offset by migration costs. Their research points to a negative selection in migration patterns since lower-skilled workers benefit the most from relocating to the US. This perspective is further corroborated by Angelucci (2015). She finds that Oportunidades, Mexico's paramount anti-poverty initiative, spurred the emigration of lower-skilled, undocumented migrants to the US.

Behrman et al. (2005) emphasize that Oportunidades' impact on schooling attendance boosts basic English proficiency and analytical competencies, both pivotal skills for success in the US labor market. Their analysis posits a non-monotonic relationship between education and migration: attaining fundamental skills heightens the propensity to migrate. Yet, further human capital accumulation diminishes this likelihood, as it renders the domestic labor market more appealing relative to its international counterpart. This pattern is also supported by Chiquiar and Hanson (2005); Hanson (2006).

To elaborate, the Mexican education system is structured into three stages:

1. Primaria or Elementary School (Grades 1-6),
2. Secundaria or Middle School (Grades 7-9), and
3. Preparatoria or High School (Grades 10-12).

Fundamental English skills are introduced during Secundaria, as detailed in Table 1. Consequently, completing Secundaria is expected to exerts a positive influence on migration decisions, while advancing from Secundaria to Preparatoria (or completing high school) is anticipated to have a negative effect on migration.

We use a decade of panel data from the Oportunidades Program to examine the impact of schooling on migration patterns. Oportunidades is a pioneering conditional cash transfer program in Mexico. The program was launched in 1997 and it randomly assigned 505 rural villages to either a treatment group (320 villages) or a control group (185 villages). Families in the treated villages received bi-monthly cash transfers, which often amounted to 20% to 30% of their household income. The transfer was contingent upon their school-age children attend school. Households in control villages had to wait for two years before receiving these benefits (Gertler, 2004).

Our study employs panel data covering the period from 1997 to 2007 to assess the influence of Oportunidades on U.S. migration among individuals who were 12 to 13 years old in 1997. This age group comprises the participants most affected by the differential schooling incentives between the treated and control groups. [20]

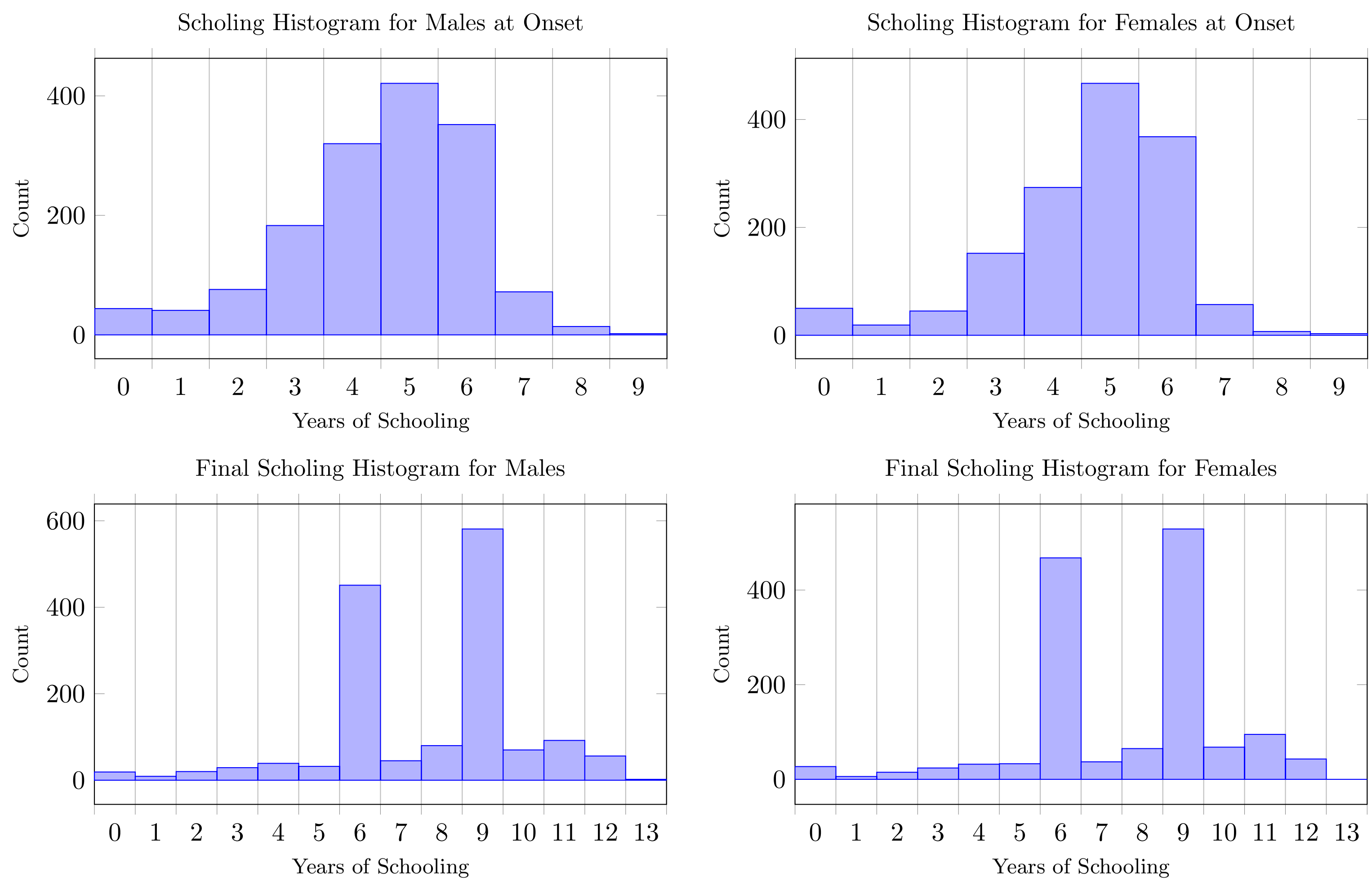
The sample comprises more than 3,000 individuals residing in impoverished rural areas. Schooling data were collected in 2003, and we utilize census data from 1997, 2003, and 2007 to examine migration patterns. Approximately 18.0% of males and 10.3% of females migrate. The majority of them move to the US between the ages of 16 and 22. Table 5 presents a statistical overview of baseline variables categorized by gender. As anticipated, baseline variables exhibit a balanced distribution across randomization arms, and none of the differences in means between the assignment groups are statistical significance.

Following our previous notation, we use $Z \in \{z_0, z_1\}$ for the randomization arms, T for years schooling, and Y for the migration outcome. Figure 1 displays the distribution of schooling at onset of the intervention in 1997 and six years after the intervention, in 2003. It is evident that Oportunidades promotes schooling. Thus, a common approach to modeling such interventions is to assume OMC, specifically, $T_i(z_0) \leq T_i(z_1)$ for all $i \in \mathcal{I}$. The OMC provides a justification for employing 2SLS regressions to examine the causal effect of Treatment T on the migration outcome Y. The results from this well-known methodology are presented in Table 3.

In our sample, Oportunidades significantly increased migration patterns and schooling attainment among males. Specifically, the intervention raised migration rates by approximately 3.0 percentage points for males. This represents an 22% increase compared to the migration probability of control group males. In the realm of education, Oportunidades led to an increase of roughly one-fourth of a school year. Table 3 also showcases the 2SLS regression where the random assignment of Oportunidades acts as an IV to evaluate the impact of education on migration. The estimated

[20]The age range was also defined in accordance to two criteria: (1) the lower boundary is set high enough to ensure that the schooling survey in 2003 measures the final schooling attainment; and (2) the upper boundary is set low enough to include individuals who were 22 years old in 2007 when the migration data was collected.

Figure 1: Histograms of Years of Schooling at the Onset and After the Intervention



This histogram displays the observed distribution of years of schooling at the onset of the intervention in 1997 and six years later, in 2003. The sample includes participants who were between the ages of 11 and 12 years old at the start of the intervention.

coefficient for males is around 0.060 and is statistically significant at 10% significance level.

The 2SLS analysis is useful to assess the overall impact of education on migration. However, coefficient evaluates a weighted average per-unit treatment effect across all individuals that increase their education when the instrument shifts from z_0 to z_1. It is difficult to relate this causal interpretation with the migration questions we seek to address. In order to advance, we evaluate a stylized model that benefits from the choice incentives and the observed patterns of school choices.

5.1 Stylized Model

We devise a stylized model that explores the tendency for education choices to cluster predominantly around the completion of Secundaria (9 years of schooling), as shown in Figure 1. Recall that completing Secundaria is a milestone in the analysis of the impact of schooling on migration since basic English skills are taught during this stage. Thus we transform the schooling variable into the index $T \in \{1, 2, 3\}$ where $T = 1$ stand for schooling less than Secundaria, and $T = 2$ stand for Secundaria competition, $T = 3$ stand for schooling beyond Secundaria.

Participants in our selected age group face three main schooling choices: (1) whether to continue studying in the year of the intervention; (2) whether to complete Secundaria; and (3) whether to continue studying beyond Secundaria. The treated group received cash transfers during all school years, which provided incentives to complete Secundaria and additional incentives to continue further studies. In contrast, the participants of the control group did not receive cash transfers during the initial years of Secundaria, influencing their decision. However, the control students who chose to continue their education received cash transfers a few years later, influencing their choice to study beyond Secundaria. The Incentive matrix corresponding to these choice incentives and its associated response matrix are presented below:

$$\boldsymbol{L} = \begin{array}{c c} \begin{array}{ccc} 1 & 2 & 3 \end{array} & \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} & \begin{array}{c} z_0 \\ z_1 \end{array} \end{array} \quad \therefore \quad \boldsymbol{R} = \begin{array}{c c} \begin{array}{ccccc} \boldsymbol{s}_{11} & \boldsymbol{s}_{22} & \boldsymbol{s}_{33} & \boldsymbol{s}_{12} & \boldsymbol{s}_{13} \end{array} & \\ \begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 2 & 3 \end{bmatrix} & \begin{array}{c} T(z_0) \\ T(z_1) \end{array} \end{array} . \tag{40}$$

The response matrix is obtained by applying Choice Rule (9) to the incentive matrix. It contains three always-takers $\boldsymbol{s}_{11}, \boldsymbol{s}_{22}, \boldsymbol{s}_{33}$, and two compliers $\boldsymbol{s}_{12}, \boldsymbol{s}_{13}$. The incentives are t-monotonic for all choices, leading to a saturated response matrix w.r.t. UMC. The incentives are also supermodular. As a result, OMC holds, but the matrix is not saturated w.r.t. OMC since choice restrictions eliminate type $[2, 3]'$.

The identification analysis stems from Theorem **T.1**. All type probabilities are (just) identified. Always-taker probabilities are given by:[21]

$$P(\boldsymbol{s}_{11}|X) = P(T = 1|z_1, X), \;\; P(\boldsymbol{s}_{22}|X) = P(T = 2|z_0, X), \quad \text{and } P(\boldsymbol{s}_{33}|X) = P(T = 3|z_0, X).$$

[21] We use $P(\boldsymbol{s}|X)$ and $P(T = t|z, X)$ as short-hand notation for $P(\boldsymbol{S} = \boldsymbol{s}|X)$ and $P(T = t|Z = z, X)$ respectively.

The probabilities for compliers are identified by:
$P(\boldsymbol{s}_{12}|X) = P(T=2|z_1,X) - P(T=2|z_0,X)$, and $P(\boldsymbol{s}_{22}|X) = P(T=3|z_1,X) - P(T=3|z_0,X)$.
There are six counterfactual outcomes that are identified. Counterfactual outcomes for always-takers are given by:
$E(Y(1)|\boldsymbol{s}_{11}) = E(Y|T=1,z_1,X), E(Y(2)|\boldsymbol{s}_{22}) = E(Y|T=2,z_0,X)$ and $E(Y(3)|\boldsymbol{s}_{33}) = E(Y|T=3,z_0,X)$.
The remaining counterfactuals are identified as LATE-type parameters:

$$\begin{aligned}
E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\}, X) &= LATE_X(\mathbf{1}[T=1]),\\
E(Y(2)|\boldsymbol{S} = \boldsymbol{s}_{12}, X) &= LATE_X(\mathbf{1}[T=2]),\\
E(Y(3)|\boldsymbol{S} = \boldsymbol{s}_{13}, X) &= LATE_X(\mathbf{1}[T=3]),\\
\text{where: } LATE_X(W) &\equiv \frac{E(Y \cdot W|Z=z_1,X) - E(Y \cdot W|Z=z_0,X)}{E(W|Z=z_1,X) - E(W|Z=z_0,X)}.
\end{aligned}$$

We are most interested in two causal effects: $E(Y(2) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$, which is the causal effect of completing Secundaria on migration, and $E(Y(3) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$, which is the causal effect of studying beyond Secundaria. Unfortunately, we cannot disentangle $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\})$ into $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$ without additional assumptions. We solve this problem of partial identification by invoking the assumption of comparable compliers:[22]

$$\text{Comparable Compliers:} \quad Y(1) \perp\!\!\!\perp \boldsymbol{S}|(T(z_0) \neq T(z_1), X). \tag{41}$$

The assumption states, that, conditioned on the compliers and on the baseline variables X, the counterfactual outcome $Y(1)$ is independent of the types. Effectively, this assumption enables the point-identification of the model by equalizing the counterfactual means for $Y(1)$ among compliers, $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12}, X) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13}, X)$.

5.2 Estimating Type Probabilities

We devise a doubly robust estimator that employs machine learning techniques to evaluate causal parameters. The method stems from the work of Navjeevan, Pinto, and Santos (2023) and has desirable properties commonly shared by this type of estimator. The method yields asymptotically normal estimators that guarantees double robustness against misspecification (Robins et al., 1995) and possesses the mixed bias property in (Chernozhukov et al., 2018). The method also benefits from variety of plug-in machine learning techniques as described in Smucler et al. (2019), Chernozhukov et al. (2022), and Chernozhukov et al. (2022).

To gain intuition, we examine the identification of type probabilities in greater detail. Let $\boldsymbol{P}_{T|X}(t) \equiv [P(T=t|Z=z_0,X), P(T=t|Z=z_1,X)]'$ be the 2×1 vector of choice probabilities across IV-values, and $\boldsymbol{P}_{T|X} \equiv [\boldsymbol{P}_{T|X}(1)', \boldsymbol{P}_{T|X}(2)', \boldsymbol{P}_{T|X}(3)']'$ be the 6×1 vector of propensity scores. Moreover, the 5×1 vector of type probabilities conditioned on X is:

$$\boldsymbol{P}_{\boldsymbol{S}|X} = [P(\boldsymbol{s}_{11}|X), P(\boldsymbol{s}_{22}|X), P(\boldsymbol{s}_{33}|X), P(\boldsymbol{s}_{12}|X), P(\boldsymbol{s}_{13}|X)]'.$$

[22] For examples of works that invoke this assumption, see, for instance, Mountjoy (2022); Navjeevan et al. (2023).

These vectors are related by the equation $\boldsymbol{P}_{T|X} = \boldsymbol{B}\boldsymbol{P}_{\boldsymbol{S}|X}$, where $\boldsymbol{B} \equiv [\boldsymbol{B}_1', \boldsymbol{B}_2', \boldsymbol{B}_3']'$ is the 8×5 binary matrix that stacks the indicator matrices $\boldsymbol{B}_t = \mathbf{1}[\boldsymbol{R} = t]$ across the treatment choices. The response matrix $\boldsymbol{R}$ is defined in (40). In this notation, we can express each of the type probabilities as a linear combination of the propensity scores:

$$P(\boldsymbol{S} = \boldsymbol{s}|X) = \boldsymbol{\nu}_{\boldsymbol{s}} \boldsymbol{P}_{Z|X} \text{ such that } \boldsymbol{\nu}_{\boldsymbol{s}} \equiv \ell_{\boldsymbol{s}}'(\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'. \tag{42}$$

The term $\boldsymbol{\nu}_{\boldsymbol{s}}$ is primary in our analysis. It is a known 6×1 vector defined as $\ell_{\boldsymbol{s}}'(\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'$, where $\ell_{\boldsymbol{s}}$ is a 5×1 canonic vector that takes value one for type $\boldsymbol{s}$ and zero otherwise. Vector $\boldsymbol{\nu}_{\boldsymbol{s}}$ can be understood as a map $\nu_{\boldsymbol{s}}(z,t)$ from the support of (Z,T) to $\mathbb{R}$. In this notation, we can rewrite equation (42) as:

$$P(\boldsymbol{S} = \boldsymbol{s}|X) = \sum_{t \in \mathcal{T}} \sum_{z \in \mathcal{Z}} \nu_{\boldsymbol{s}}(z,t) P(T = t|Z = z, X). \tag{43}$$

To construct the doubly robust estimator, we represent the type probability as the expectation of a function κ such that $P(\boldsymbol{S} = \boldsymbol{s}) = E(\kappa_{\boldsymbol{s}}(T,Z,X))$.[23] The doubly robust estimator is based on the following the orthogonal score representation of type probabilities:

$$P(\boldsymbol{S} \; = \boldsymbol{s}) = \sum_{t \in \mathcal{T}} E_{ZX}\left(\kappa_{\boldsymbol{s}}(t,Z,X) \cdot \left(\mathbf{1}[T=t] - P(T=t|Z,X)\right)\right) + \sum_{t \in \mathcal{T}} E_X\left(\sum_{z \in \mathcal{Z}} \nu_{\boldsymbol{s}}(z,t) P(T=t|Z=z,X)\right),$$

where $E_{ZX}(\cdot)$ is an expectation over the joint distribution of (Z,X) and $E_X(\cdot)$ is an expectation over X. The identifying moment condition has two nuisance parameters, the function $\kappa_{\boldsymbol{s}}(t,Z,X)$ and the propensity score $P(T=t|Z,X)$. We assess these nuances via plug-in estimators, that is, we evaluate the propensity score $P(T=t|Z,X)$ by $\boldsymbol{h}(Z,X)\boldsymbol{\beta}_t$, and the kappa function $\kappa_{\boldsymbol{s}}(t,Z,X)$ by $\boldsymbol{h}(Z,X)\boldsymbol{\gamma}_{\boldsymbol{s},t}$, where $\boldsymbol{\beta}_t, \boldsymbol{\gamma}_t$ are p-dimensional linear coefficients and $\boldsymbol{h}(Z,X) = [b_1(Z,X), ..., b_p(Z,X)]'$ denotes a p-dimensional vector of function of (Z,X) including all the pairwise interactions of these variables. In our application, $\boldsymbol{h}(Z,X)$ comprises X, Z, and their interaction. Appendix A.13 presents a detailed description of the estimation algorithm.

In Table 4, we present the estimated probabilities for each type. The aggregate probability for the always-takers is approximately 0.90, indicating that 90% of the sample comprises students who persist with their schooling choice towards Secundaria irrespective of their allocation to either treatment or control groups. The probability for type $\boldsymbol{s}_{11}$ stands at around 0.43, suggesting that nearly half of the sample consists of students who do not complete Secundaria, regardless of the incentives from Oportunidades. The probability associated with type $\boldsymbol{s}_{22}$ is close to 0.34, denoting that a third of the sample consistently chooses to finalize their Secundaria. Lastly, the probability for type $\boldsymbol{s}_{33}$ is approximately 0.13, implying that a mere 13% of the students opt to pursue education beyond Secundaria, irrespective of receiving the Oportunidades incentives or not.

The sum of the probabilities for compliers, $\boldsymbol{s}_{12}$ and $\boldsymbol{s}_{13}$, totals 0.093. This means that about 9% of the students change their choice towards completing Secundaria when the incentives provided

[23] See Navjeevan, Pinto, and Santos (2023) for a in-depth discussion of the rationale of this approach.

by Oportunidades are available. The majority of these students, about 7%, consists of participants of type $\boldsymbol{s}_{12}$ who shift for not completing Secundaria to completing Secundaria. A smaller share of sample, about 2%, comprises compliers that change their student decision from not completing Secundaria when assigned to control to studying beyond Secondaria when assigned to the treatment.

5.3 Estimating Causal Effects

We now describe the doubly robust estimators used to evaluate the counterfactual outcomes and the causal effects of our model. Our discussion mirrors the approach we took when investigating type probabilities. To provide more insight, we examine the identification of counterfactual outcomes through moment conditions.

Let $\boldsymbol{E}_{Y|X}(t) \equiv [E(Y \cdot \mathbf{1}[T=t]|Z=z_0,X), E(Y \cdot \mathbf{1}[T=t]|Z=z_1,X)]'$ be the 2×1 vector of conditional outcome moments. As mentioned, $\boldsymbol{B}_t = \mathbf{1}[\boldsymbol{R}=t]$ denotes the binary matrix that indicates which elements in the response matrix $\boldsymbol{R}$ in (40) takes value $t \in \{1,2,3\}$. We use this notation to express the identified counterfactual outcomes – $E(Y(1)|\boldsymbol{s}_{11})$, $E(Y(2)|\boldsymbol{s}_{22})$, $E(Y(3)|\boldsymbol{s}_{33})$ $E(Y(2)|\boldsymbol{S}=\boldsymbol{s}_{12},X)$ and $E(Y(3)|\boldsymbol{S}=\boldsymbol{s}_{13},X)$ – in the following fashion:

$$E(Y(t)|\boldsymbol{S}=\boldsymbol{s})P(\boldsymbol{S}=\boldsymbol{s}|X) = \boldsymbol{\nu}_{\boldsymbol{s},t}\boldsymbol{E}_{Z|X} \text{ such that } \boldsymbol{\nu}_{\boldsymbol{s},t} \equiv \ell'_{\boldsymbol{s}}\boldsymbol{B}'_t(\boldsymbol{B}_t\boldsymbol{B}'_t)^{-1}, \tag{44}$$

where $\ell_{\boldsymbol{s}}$ is a 5×1 canonic vector that indicates type $\boldsymbol{s}$. Similar to our analysis of type probabilities, we can express $\boldsymbol{\nu}_{\boldsymbol{s},t}$ as a function $\nu_{\boldsymbol{s},t}(z)$ from the support of Z to $\mathbb{R}$. In this notation, we can rewrite equation (44) as:

$$E(Y(t)\mathbf{1}[\boldsymbol{S}=\boldsymbol{s}]|X)P(\boldsymbol{S}=\boldsymbol{s}|X) = \sum_{z\in\mathcal{Z}} \nu_{\boldsymbol{s},t}(z)E(Y\cdot\mathbf{1}[T=t]|Z=z,X). \tag{45}$$

It is also worth noting that the response types probability associated with the identified outcome counterfactuals can be identified as:

$$P(\boldsymbol{S}=\boldsymbol{s}|X) = \sum_{z\in\mathcal{Z}} \nu_{\boldsymbol{s},t}(z)E(\mathbf{1}[T=t]|Z=z,X). \tag{46}$$

The doubly robust estimator comprises the joint evaluation of the expectation $E(Y(t)\mathbf{1}[\boldsymbol{S}=\boldsymbol{s}])$ in (45), the probability $P(\boldsymbol{S}=\boldsymbol{s}|X)$ and then taking the ratio of these estimates. Note that both problems are related since they are associated with the same identification function $\nu_{\boldsymbol{s},t}(z)$. The estimator for $E(Y(t)\mathbf{1}[\boldsymbol{S}=\boldsymbol{s}])$ is based on the following orthogonal score:

$$E(Y(t)\mathbf{1}[\boldsymbol{S}=\boldsymbol{s}]) = E_{ZX}\Big(Y\kappa_{\boldsymbol{s},t}(Z,X)\cdot\big(Y\mathbf{1}[T=t]-E(Y\mathbf{1}[T=t]|Z,X)\big)\Big) + E_X\left(\sum_{z\in\mathcal{Z}}\nu_{\boldsymbol{s},t}(z)E(Y\cdot\mathbf{1}[T=t]|Z=z,X)\right).$$

The function kappa is such that $E(Y\kappa_{\boldsymbol{s},t}(Z,X)) = E(Y(t)\mathbf{1}[\boldsymbol{S}=\boldsymbol{s}])$ and $E(\kappa_{\boldsymbol{s},t}(Z,X)) = P(\boldsymbol{S}=\boldsymbol{s})$. The estimator contains three nuance parameters: the propensity score $P(T=t|Z,X)$ is estimated as by $\boldsymbol{h}(Z,X)\boldsymbol{\beta}_t$, the outcome expectation $E(Y\cdot\mathbf{1}[T=t]|Z=z,X)$ by $\boldsymbol{h}(Z,X)\boldsymbol{\theta}_t$, and the kappa function $\kappa_{\boldsymbol{s},t}(t, Z, X)$ by $\boldsymbol{h}(Z, X)\boldsymbol{\gamma}_{\boldsymbol{s},t}$, where $\boldsymbol{\beta}_t, \boldsymbol{\theta}_t, \boldsymbol{\gamma}_t$ are p-dimensional linear coefficients and $\boldsymbol{h}(Z, X)$ comprises X, Z, and their interaction. The steps of the estimator are closely related to the estimation of type probabilities. Appendix A.14 describes the estimation algorithm in detail.

Recall that we aim to evaluate two causal effects: $E(Y(2) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(2) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$. The procedure outlined in Appendix A.14 is tailored to estimate any counterfactual outcome that is identified according to the response matrix $\boldsymbol{R}$ (40). These include $E(Y(2)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(3)|\boldsymbol{S} = \boldsymbol{s}_{13})$. The procedure could also be used to evaluate $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\})$, since it is also identified. The procedure however is not suitable to evaluate $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$ separately.

The additional assumption of comparable compliers (41) enable us to disentangle $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\})$ into $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$ and $E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$. The assumption implies that $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\}|X) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12}|X)$ and $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\}|X) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13}|X)$. Note however that this assumption does not imply the unconditional equality $E(Y(1)|\boldsymbol{S} \in \{\boldsymbol{s}_{12}, \boldsymbol{s}_{13}\}) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12}) = E(Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$ because the distribution of baseline variables X may differ across types $\boldsymbol{s}_{12}$ and $\boldsymbol{s}_{13}$. The modification of the procedure is necessary to account for the difference in the distribution of baseline variables X between types. Navjeevan, Pinto, and Santos (2023) solve the same problem in a different setting involving the mediation analysis of a choice model containing seven types. We apply their solution to our setting. The estimation procedure is more complex than the one used to the previous counterfactual outcomes. See Appendix A.15 for a detailed description of the estimation algorithm.

Table 5 presents the causal effects of our model conditioned on different sets of baseline variables. The first panel of the table presents the estimates for $E(Y(2) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{12})$, which evaluates the casual effect of completing Secundaria on migration for the subset of compliers that change from not completing Secundaria to completing it when the incentives of Oportunidades are available. These compliers account for about 7% of the sample. We find that completing Secundaria has a substantial impact on the decision to migrate. The causal effect is about 0.48 and the estimates are statistically significant at 10% significance level.

The second panel of the table displays the estimates for $E(Y(3) - Y(1)|\boldsymbol{S} = \boldsymbol{s}_{13})$, which is the causal effect of changing education attainment from not completing Secundaria to study beyond Secundaria on migration. This effect is associated to 2% of the participants. It comprises the subset of compliers that decide to study beyond Secundaria due to the incentives offered by Oportunidades. We find the effect to be negative, relatively small, and not statistically significant. The point estimate of the effect ranges from -0.10 to -0.20 when conditioned on baseline variables.

The main feature of this empirical exercise is the use of incentive analysis to assess the question of whether schooling has a non-monotonic effect on the decision to migrate to the US. We focus on the age range most likely to respond to the schooling incentives offered by Oportunidades. Our stylized model enables us to characterize five types that are driven by economic behavior. We are able to evaluate the share of the participants that do respond to Oprtunidades' incentives and evaluate the causal effects of schooling on migration for those who respond to the incentives. We find compelling evidence that completing Secundaria increases the likelihood of migration. Our key empirical finding however is the difference between the causal effects. While completing Secundaria has a strong effect on the propensity to migrate, studying beyond Secundaria does not. These findings corroborate the hypothesis of several works suggesting a negative and non-monotonic selection of migrants regarding education (Behrman et al., 2005; Borjas, 1987; Chiquiar and Hanson, 2005).

Table 1: Skills taught by School Level
Biggest expected effect on migration moving from primary to secondary

Primaria (Grades 1-6)	Secundaria (grades 7-9)	Preparatoria (grades 10-12)
Basic Reading and writing skills	Intermediate Reading and writing skills	Advanced reading and writing skills
Basic mathematical skills	Intermediate mathematical skills	Advanced mathematical skills
To search for information	Work-oriented skills in workshops	Specialized technical-level skills for work
	(carpentry, plumbing and electricity, cooking, etc)	Critical thinking
To follow instructions	**Essential skills for communication in English**	Vocational guidance for jobs in Mexico
Basic comprehension of natural	Civic and social participation	Social skills
and social environments	Self-learning and Reproductive health	Time management

Table 2: Statistical Description of Baseline Variables

	Males			Females		
	Treated Mean	Control Mean	Diff. Means	Treated Mean	Control Mean	Diff. Means
Age at Onset	11.930	11.929	0.000	11.872	11.909	-0.036
s.e.	0.572	0.553	0.028	0.567	0.584	0.029
Family Speaks Indigenous Language	0.417	0.469	-0.052	0.409	0.425	-0.016
s.e.	0.493	0.499	0.025	0.492	0.495	0.025
Household Assets Index	624.27	616.39	7.880	618.81	625.23	-6.428
s.e.	88.546	98.092	4.685	89.707	91.044	4.542
Number of Household Members	7.581	7.476	0.105	7.625	7.594	0.031
s.e.	2.195	2.097	0.106	2.118	2.049	0.104
Household Members Younger than 17	4.739	4.702	0.037	4.789	4.767	0.022
s.e.	1.784	1.736	0.087	1.705	1.702	0.085
County USA Migration Index	-0.155	-0.170	0.015	-0.115	-0.183	0.068
s.e.	0.843	0.919	0.045	0.878	0.908	0.046
Home Ownership	0.969	0.951	0.018	0.971	0.955	0.016
s.e.	0.174	0.216	0.010	0.167	0.207	0.010
Schooling at Onset	4.546	4.407	0.140	4.535	4.611	-0.076
s.e.	1.618	1.630	0.081	1.577	1.596	0.080
Sample Size	1027	674		1048	645	

This columns of this table presents the statistical description of baseline variables by gender. The first row associated to each variable displays the treatment mean, control mean and the mean difference for males and females. The second row displays the standard deviation of the treated and control means and the standard error for the difference-in-means estimator.

Table 3: Standard 2SLS Analysis on the Effect of Oportunidades on Schooling and Migration

	Males				Females			
	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 1**	**Model 2**	**Model 3**	**Model 4**
Effects of the Oportunidades Intervention								
Migration	**0.037**	**0.033**	0.029	**0.031**	0.005	-0.002	-0.001	0.001
s.e.	0.020	0.019	0.018	0.018	0.016	0.016	0.015	0.015
p-val	0.061	0.073	0.113	0.094	0.753	0.873	0.937	0.965
Schooling	**0.502**	**0.522**	**0.533**	**0.521**	0.022	0.044	0.050	0.076
s.e.	0.121	0.122	0.122	0.122	0.126	0.128	0.128	0.128
p-val	0.000	0.000	0.000	0.000	0.860	0.733	0.695	0.554
Effects of Schooling on Migration								
2SLS	**0.073**	**0.064**	0.055	0.059	0.227	-0.057	-0.024	0.009
s.e.	0.043	0.039	0.037	0.038	1.478	0.390	0.313	0.205
p-val	0.092	0.099	0.135	0.118	0.878	0.885	0.938	0.965
OLS	-0.005	0.000	0.002	0.002	-0.001	-0.001	-0.001	-0.001
s.e.	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003
p-val	0.259	0.980	0.663	0.549	0.803	0.719	0.801	0.792

The table comprises four panels. The first panel displays the causal effects of the Oportunidades on migration. The second panel displays the effects on Schooling (measured in 2003), it is also the first stage for the 2SLS on the third panes, that evaluates the causal effect of Schooling on migration using the Oportunidades random assignment as an instrumental variable for schooling. Each panel presents the estimates by gender across four models that differ in terms of conditioning variables. Model 1 does not use conditioning variables. Model 2 employs age at onset and county migration index. Model 3 adds family characteristics: family members speak indigenous language, number of household members, and number of teenagers. Model 4 includes household assets and house ownership. Estimates consists on the effect, its standard error and the double-sided *p*-value associated with inference that tests if the effect is equal to zero. All estimates are based on the standard OLS and 2SLS regressions. Inference employs clustered errors.

Table 4: Type Probabilities and Causal Effects for Males

Type Probabilities	**Model 1**	**Model 2**	**Model 3**	**Model 4**
$P(\boldsymbol{S} = \boldsymbol{s}_{11})$	**0.436**	**0.436**	**0.435**	**0.436**
s.e.	0.016	0.016	0.016	0.016
p-val	0.000	0.000	0.000	0.000
$P(\boldsymbol{S} = \boldsymbol{s}_{22})$	**0.337**	**0.338**	**0.337**	**0.340**
s.e.	0.019	0.019	0.019	0.019
p-val	0.000	0.000	0.000	0.000
$P(\boldsymbol{S} = \boldsymbol{s}_{33})$	**0.133**	**0.133**	**0.130**	**0.131**
s.e.	0.014	0.014	0.014	0.014
p-val	0.000	0.000	0.000	0.000
$P(\boldsymbol{S} = \boldsymbol{s}_{44})$	**0.071**	**0.071**	**0.070**	**0.068**
s.e.	0.024	0.024	0.025	0.025
p-val	0.004	0.004	0.005	0.007
$P(\boldsymbol{S} = \boldsymbol{s}_{55})$	0.022	0.023	0.026	0.026
s.e.	0.018	0.018	0.018	0.018
p-val	0.206	0.181	0.139	0.149

This table presents the estimates of type probabilities according to the doubly robust orthogonal score estimator described in this section. Estimates are presents for four models that vary in the set of baseline variables X that we seek to condition on. Model 1 does not use baseline variables. Model 2 employs age at onset and county migration index. Model 3 adds family characteristics: family members speak indigenous language, number of household members, and number of teenagers. Model 4 includes household assets and house ownership. Estimates consists on the probability, its standard error and the two-sided p-value associated with inference that tests if the effect is equal to zero. Standard errors are computed using the multiplier bootstrap method.

Table 5: Causal Effects for Males

Causal Effects	**Model 1**	**Model 2**	**Model 3**	**Model 4**
$E(Y(2) - Y(1) \mid \boldsymbol{S} = \boldsymbol{s}_{21})$	**0.480**	**0.487**	**0.472**	**0.503**
s.e.	0.265	0.261	0.261	0.287
p-val	0.070	0.062	0.071	0.079
$E(Y(3) - Y(1) \mid \boldsymbol{S} = \boldsymbol{s}_{31})$	-0.001	-0.118	-0.135	-0.194
s.e.	0.274	0.320	0.310	0.358
p-val	0.998	0.713	0.662	0.587

This table presents the estimates of the causal effects for males. The estimates are obtained according to the doubly robust orthogonal score estimator described in this section. The estimates comprise four models that differ in terms of the set of baseline variables we seek to control for. Model 1 does not include baseline variables. Model 2 employs age at onset and county migration index. Model 3 adds family characteristics: family members speak indigenous language, number of household members, and number of teenagers. Model 4 includes household assets and house ownership. Estimates consists on the effect, its standard error and the two-sided p-value associated with inference that tests if the effect is equal to zero. Standard errors are computed using the multiplier bootstrap method.

6 Summary and Conclusions

This paper has provided a fresh perspective on the identification of causal effects in instrumental variable (IV) models within economics. While the conventional approach has focused on developing novel monotonicity or separability conditions, we have proposed a departure from this mindset.

Our approach is rooted in the utilization of economic incentives and classical economic behavior to identify causal parameters in IV models with multiple choices and categorical instruments. We introduced a flexible framework based on revealed preference analysis, which translates choice incentives into identification conditions. This method has several notable advantages, most notably its independence from statistical or functional form assumptions. Instead, identification conditions arise organically from fundamental economic principles applied to choice incentives, enhancing both credibility and understanding.

Moreover, our framework is versatile enough to accommodate a wide range of non-trivial identification assumptions, making it applicable in scenarios where traditional IV assumptions may not hold. We have demonstrated its flexibility by examining well-established examples of choice incentives in the policy evaluation literature, showcasing its adaptability to real-world empirical research.

We employ our analytical framework to investigate the migration patterns of impoverished Mexican households to the US. A substantial literature on migration investigates the relationship between education attainment and the likelihood of migration. Seminal work of Borjas (1987, 1994) suggests a negative selection in which those with lowest education benefit the most from moving to the US. On the other hand, Behrman et al. (2005); Chiquiar and Hanson (2005); Hanson (2006) posits a non-monotonic relationship between education and migration, where the fundamental skills such as basic English proficiency taught in Secundaria (middle school) increase the propensity to migrate while additional education reduces migration.

We utilize data from Oportunidades, the largest and most significant anti-poverty program in Mexico, to examine whether schooling has a non-monotonic impact on the decision to migrate to the United States. Employing our incentive framework, we identify two causal effects of education on migration for students responding to the schooling incentives provided by Oportunidades.

Specifically, we assess the impact of completing Secundaria and the effects of pursuing education beyond this level. Our findings provide compelling evidence that completing Secundaria increases the likelihood of migration, whereas advancing schooling beyond middle school has a negative effect on migration. We estimate our model using novel machine learning techniques that assure double robustness of our estimates.

In the broader context of economic research, this paper contributes to the growing body of literature that leverages revealed preference analysis to enhance the identification of causal effects in IV models. Our approach offers a valuable tool for economists grappling with identification issues in diverse and non-standard empirical settings. We make the case that combining economic incentives and classical behavior strengthen the foundations of IV analysis and empowers researchers with a useful tool to evaluate such models.

References

Aizer, A. and J. J. Doyle (2015). Juvenile incarceration, human capital, and future crime: Evidence from randomly assigned judges. *The Quarterly Journal of Economics 130*(2), 759–804.

Aliprantis, D. (2012). Compulsory schooling laws and educational attainment. *Journal of Educational and Behavioral Statistics 37*, 316–338.

Andresen, M. E. and M. Huber (2021). Instrument-based estimation with binarised treatments: issues and tests for the exclusion restriction. *The Econometrics Journal 24*(3), 536–558.

Angelucci, M. A. (2015). Migration and financial constraints: Evidence from mexico. *The Review of Economics and Statistics 97*(1), 224–228.

Angrist, J. D., K. Graddy, and G. W. Imbens (2000, July). The interpretation of instrumental variables estimators in simultaneous equations models with an application to the demand for fish. *Review of Economic Studies 67*(3), 499–527.

Angrist, J. D. and G. W. Imbens (1995, June). Two-stage least squares estimation of average causal effects in models with variable treatment intensity. *Journal of the American Statistical Association 90*(430), 431–442.

Arteaga, C. (2021, 10). Parental incarceration and children's educational attainment. *The Review of Economics and Statistics*, 1–45.

Barua, R. and K. Lang (2016). School entry, educational attainment, and quarter of birth: A cautionary tale of a local average treatment effect. *Journal of Human Capital 10*(3), 347–376.

Behrman, J., P. Sengupta, and P. Todd (2005). Progressing through progresa: An impact assessment of a school subsidy experiment in rural mexico. *Economic Development and Cultural Change 54*(1), 237–275.

Bhuller, M., L. K. V. Dahl, Gordon B., and M. Mogstad (2020). Incarceration, recidivism, and employment. *Journal of Political Economy 128*(4), 1269–1324.

Black, S. E., P. J. Devereux, and K. G. Salvanes (2005, March). Why the apple doesn't fall far: Understanding intergenerational transmission of human capital. *American Economic Review 95*(1), 437–449.

Borjas, G. J. (1987). Self-selection and the earnings of immigrants. *The American Economic Review 77*(4), 531–553.

Borjas, G. J. (1994). The economics of immigration. *Journal of Economic Literature 32*(4), 1667–1717.

Brinch, C. N., M. Mogstad, and M. Wiswall (2017). Beyond late with a discrete instrument. *Journal of Political Economy 125*(4), 985–1039.

Cameron, S. V. and J. J. Heckman (1998, April). Life cycle schooling and dynamic selection bias: Models and evidence for five cohorts of American males. *Journal of Political Economy 106*(2), 262–333.

Carneiro, P., K. Hansen, and J. J. Heckman (2003, May). Estimating distributions of treatment effects with an application to the returns to schooling and measurement of the effects of uncertainty on college choice. *International Economic Review 44*(2), 361–422.

Carneiro, P., J. J. Heckman, and E. J. Vytlacil (2011, September). Estimating marginal returns to education. *American Economic Review 101*(6), 2754–2781.

Chernozhukov, V., D. Chetverikov, M. Demirer, E. Duflo, C. Hansen, W. Newey, and J. Robins (2018). Double/debiased machine learning for treatment and structural parameters.

Chernozhukov, V., J. C. Escanciano, H. Ichimura, W. K. Newey, and J. M. Robins (2022). Locally robust semiparametric estimation. *Econometrica 90*(4), 1501–1535.

Chernozhukov, V., W. K. Newey, and R. Singh (2022). Automatic debiased machine learning of causal and structural effects. *Econometrica 90*(3), 967–1027.

Chiquiar, D. and G. Hanson (2005). International migration, self-selection, and the distribution of wages: Evidence from mexico and the united states. *Journal of Political Economy 113*(2), 239–281.

Cunha, F., J. J. Heckman, and S. Navarro (2007, November). The identification and economic content of ordered choice models with stochastic cutoffs. *International Economic Review 48*(4), 1273–1309.

Dahl, C. M., M. Huber, and G. Mellace (2017). It's never too late: A new look at local average treatment effects with or without defiers. *Discussion Papers on Business and Economics*.

de Chaisemartin, C. (2017). Tolerating defiance? local average treatment effects without monotonicity. *Quantitative Economics 8*(2), 367–396.

Feller, A., T. Grindal, L. Miratrix, and L. C. Page (2016). Compared to what? variation in the impacts of early childhood education by alternative care type. *The Annals of Applied Statistics 10*(3), 1245–1285.

Finkelstein, A., S. Taubman, B. Wright, M. Bernstein, J. Gruber, J. P. Newhouse, H. Allen, K. Baicker, and O. H. S. Group (2012, 07). The oregon health insurance experiment: Evidence from the first year*. *The Quarterly Journal of Economics 127*(3), 1057–1106.

Gertler, P. (2004, May). Do conditional cash transfers improve child health? evidence from progresa's control randomized experiment. *American Economic Review 94*(2), 336–341.

Hanson, G. H. (2006, December). Illegal migration from mexico to the united states. *Journal of Economic Literature 44*(4), 869–924.

Heckman, J. and R. Pinto (2013). Causal analysis after haavelmo. *Economic Theory Forthcoming.*

Heckman, J. and R. Pinto (2018). Unordered monotonicity. *Econometrica 86*, 1–35.

Heckman, J. and R. Pinto (2022). Econometric causality. *Annual Reviews of Economics 14*, 893–923.

Heckman, J. J. (2010). Building bridges between structural and program evaluation approaches to evaluating policy. *Journal of Economic Literature 48*(2), 356–398.

Heckman, J. J. and S. Urzúa (2010, May). Comparing IV with structural models: What simple IV can and cannot identify. *Journal of Econometrics 156*(1), 27–37.

Heckman, J. J., S. Urzua, and E. Vytlacil (2006). Understanding instrumental variables in models with essential heterogeneity. *The Review of Economics and Statistics 88*(3), 389–432.

Heckman, J. J., S. Urzúa, and E. J. Vytlacil (2008). Instrumental variables in models with multiple outcomes: The general unordered case. *Annales d'Economie et de Statistique 91–92*, 151–174.

Heckman, J. J. and E. J. Vytlacil (1999, April). Local instrumental variables and latent variable models for identifying and bounding treatment effects. *Proceedings of the National Academy of Sciences 96*(8), 4730–4734.

Heckman, J. J. and E. J. Vytlacil (2005, May). Structural equations, treatment effects and econometric policy evaluation. *Econometrica 73*(3), 669–738.

Heckman, J. J. and E. J. Vytlacil (2007a). Econometric evaluation of social programs, part I: Causal models, structural models and econometric policy evaluation. In J. J. Heckman and E. E. Leamer (Eds.), *Handbook of Econometrics*, Volume 6B, Chapter 70, pp. 4779–4874. Amsterdam: Elsevier B. V.

Heckman, J. J. and E. J. Vytlacil (2007b). Econometric evaluation of social programs, part II: Using the marginal treatment effect to organize alternative economic estimators to evaluate social programs, and to forecast their effects in new environments. In J. J. Heckman and E. E. Leamer (Eds.), *Handbook of Econometrics*, Volume 6B, Chapter 71, pp. 4875–5143. Amsterdam: Elsevier B. V.

Holland, P. W. (1986, December). Statistics and causal inference. *Journal of the American Statistical Association 81*(396), 945–960.

Huber, M., L. Laffers, and G. Mellace (2017). Sharp iv bounds on average treatment effects on the treated and other populations under endogeneity and noncompliance. *Journal of Applied Econometrics 32*(1), 56–79.

Huber, M. and G. Mellace (2012). Relaxing monotonicity in the identification of local average treatment effects. Unpublished manuscript, Department of Economics, University of St. Gallen.

Huber, M. and G. Mellace (2015). Testing instrument validity for late identification based on inequality moment constraints. *Review of Economics and Statistics 97*, 398–411.

Hull, P. (2018). Isolateing: Identifying counterfactual-specific treatment effects with cross-stratum comparisons. *Umpublished Manuscript*.

Imbens, G. W. and J. D. Angrist (1994, March). Identification and estimation of local average treatment effects. *Econometrica 62*(2), 467–475.

Imbens, G. W. and D. B. Rubin (1997, October). Estimating outcome distributions for compliers in instrumental variables models. *Review of Economic Studies 64*(4), 555–574.

Kamat, V. (2021). Identifying the effects of a program offer with an application to head start. *Umpublished Manuscrip*.

Kane, T. J. and C. E. Rouse (1995, June). Labor-market returns to two- and four-year college. *American Economic Review 85*(3), 600–614.

Kirkeboen, L. J., E. Leuven, and M. Mogstad (2016). Field of study, earnings, and self-selection. *The Quarterly Journal of Economics 131*(3), 1057–1111.

Klein, T. J. (2010). Heterogeneous treatment effects: Instrumental variables without monotonicity? *Journal of Econometrics 155*, 99–116.

Kline, P. and M. Tartari (2016, April). Bounding the labor supply responses to a randomized welfare experiment: A revealed preference approach. *American Economic Review 106*(4), 972–1014.

Kline, P. and C. Walters (2016a). Evaluating public programs with close substitutes: The case of Head Start. Quarterly Journal of Economics *131*(4), 1795–1848.

Kline, P. and C. R. Walters (2016b, 07). Evaluating public programs with close substitutes: The case of head start. *The Quarterly Journal of Economics 131*(4), 1795–1848.

Lange, F. (2011). The role of education in complex health decisions: Evidence from cancer screening. *Journal of Health Economics 30*(1), 43–54.

Lee, S. and B. Salanié (2018). Identifying effects of multivalued treatments. *Econometrica 86*, 1939–1963.

Magnus, J. R. and H. Neudecker (1999). *Matrix Differential Calculus with Applications in Statistics and Econometrics* (2 ed.). Wiley.

Mogstad, M., A. Santos, and A. Torgovitsky (2018). Using instrumental variables for inference about policy relevant treatment parameters. *Econometrica 86*(5), 1589–1619.

Mogstad, M. and A. Torgovitsky (2018). Identification and extrapolation of causal effects with instrumental variables. *Annual Review of Economics 2*, 577–613.

Mogstad, M., A. Torgovitsky, and C. R. Walters (2021a). The causal interpretation of two-stage least squares with multiple instrumental variables. *American Economic Review 111*(11), 3663–3698.

Mogstad, M., A. Torgovitsky, and C. R. Walters (2021b). Policy evaluation with multiple instrumental variables. *Forthcoming Journal of Econometrics*.

Mogstad, M. and M. Wiswall (2016). Testing the quantity-quality model of fertility: Linearity, marginal effects, and total effects. Forthcoming, *Quantitative Economics*.

Mountjoy, J. (2021). Community colleges and upward mobility. *Umpublished Manuscript*.

Mountjoy, J. (2022). Community colleges and upward mobility. *American Economic Review 112*(8), 2580–2630.

Navjeevan, M., R. Pinto, and A. Santos (2023). Identification and estimation in a class of potential outcomes models.

Pinto, R. (2022). Beyond intention to treat: Using the incentives in moving to opportunity to identify neighborhood effects. *NBER Working Paper*.

Richter, M. K. (1971). Rational choice. In *Preferences, Utility, and Demand: A Minnesota Symposium* (1 ed.), Volume 1, Chapter 2, pp. 29–58. New York: Harcourt, Brace, Jovanovich.

Robins, J. M., A. Rotnitzky, and L. P. Zhao (1995). Analysis of semiparametric regression models for repeated outcomes in the presence of missing data. *Journal of the American Statistical Association 90*(429), 106–121.

Rose, E. K. and Y. Shem-Tov (2021). On recoding ordered treatments as binary indicators. *NBER Working Paper*.

Rose, E. K. and Y. Shem-Tov (2023). On recoding ordered treatments as binary indicators. *Working Paper*.

Rubin, D. B. (1978, January). Bayesian inference for causal effects: The role of randomization. *Annals of Statistics 6*(1), 34–58.

Ryser, H. (1957). Combinatorial properties of matrices of zeros and ones. *Pacific J. Math. 7*, 1073–1082.

Small, D. S. and Z. Tan (2007). A stochastic monotonicity assumption for the instrumental variables method. *Technical report, Department of Statistics, Wharton School, University of Pennsylvania.*.

Smucler, E., A. Rotnitzky, and J. M. Robins (2019). A unifying approach for doubly-robust ℓ_1 regularized estimation of causal contrasts. *arXiv preprint arXiv:1904.03737*.

Vytlacil, E. J. (2002, January). Independence, monotonicity, and latent index models: An equivalence result. *Econometrica 70*(1), 331–341.

Vytlacil, E. J. (2006, August). Ordered discrete-choice selection models and local average treatment effect assumptions: Equivalence, nonequivalence, and representation results. *Review of Economics and Statistics 88*(3), 578–581.