Online Appendix for:

Beyond Intention to Treat: Using the Incentives in *Moving to Opportunity* to Identify Neighborhood Effects

A Mathematical Proofs

A.1 Proof of Proposition P.1

Let the response vector be $\boldsymbol{S} = [T(z_1), ..., T(z_n)]'$ where $supp(Z) = \{z_1, ...z_n\}$. Note that the treatment T can be expressed as $T = [\mathbf{1}[Z = z_1], ..., \mathbf{1}[Z = z_1]]\boldsymbol{S}$. This implies that T depends only on Z when conditioned on $\boldsymbol{S}$. Moreover, T is deterministic given Z and $\boldsymbol{S}$. The Exogeneity Condition 2 states that $Z \perp\!\!\!\perp (Y(t), T(z), Y(z))$. This assumption implies the following relationships:

$$Z \perp\!\!\!\perp \boldsymbol{S} \tag{63}$$

$$Y(t) \perp\!\!\!\perp (Z, T)|\boldsymbol{S} \tag{64}$$

Relationship (63) is due to $Z \perp\!\!\!\perp T(z)$. Relationship (64) arises from $Y(t) \perp\!\!\!\perp Z|T(z)$ and the fact that T is a function of Z when conditioned on $\boldsymbol{S}$. Finally, the Exclusion Restriction (1) enable us to express the observed outcome as:

$$Y = \sum_{t \in supp(T)} \mathbf{1}[T = t] \cdot Y(t). \tag{65}$$

The derivation of the equation (8) is displayed below:

$$\begin{aligned}
E(Y|Z = z, T = t) &= \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} E(Y|Z = z, T = t, \boldsymbol{S} = \boldsymbol{s})P(\boldsymbol{S} = \boldsymbol{s}|T = t, Z = z) \\
&= \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} E(Y|Z = z, T = t, \boldsymbol{S} = \boldsymbol{s})\frac{P(T = t|\boldsymbol{S} = \boldsymbol{s}, Z = z)P(\boldsymbol{S} = \boldsymbol{s}|Z = z)}{P(T = t|Z = z)} \\
\Rightarrow E(Y|Z = z, T = t)P(T = t|Z = z) &= \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} E(Y|Z = z, T = t, \boldsymbol{S} = \boldsymbol{s})P(T = t|\boldsymbol{S} = \boldsymbol{s}, Z = z)P(\boldsymbol{S} = \boldsymbol{s}|Z = z) \\
&= \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} E(Y|Z = z, T = t, \boldsymbol{S} = \boldsymbol{s})P(T = t|\boldsymbol{S} = \boldsymbol{s}, Z = z)P(\boldsymbol{S} = \boldsymbol{s}) \\
&= \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} E(Y|Z = z, \boldsymbol{S} = \boldsymbol{s})\mathbf{1}[T = t|\boldsymbol{S} = \boldsymbol{s}, Z = z]P(\boldsymbol{S} = \boldsymbol{s}) \\
&= \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} \mathbf{1}[T = t|\boldsymbol{S} = \boldsymbol{s}, Z = z]E(Y|Z = z, T = t, \boldsymbol{S} = \boldsymbol{s})P(\boldsymbol{S} = \boldsymbol{s}) \\
&= \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} \mathbf{1}[T = t|\boldsymbol{S} = \boldsymbol{s}, Z = z]E(Y(t)|Z = z, T = t, \boldsymbol{S} = \boldsymbol{s})P(\boldsymbol{S} = \boldsymbol{s}) \\
&= \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} \mathbf{1}[T = t|\boldsymbol{S} = \boldsymbol{s}, Z = z]E(Y(t)|\boldsymbol{S} = \boldsymbol{s})P(\boldsymbol{S} = \boldsymbol{s})
\end{aligned}$$

The first equality applies the law of iterated expectations to the expectation $E(Y|Z = z, T = t)$. The second equality uses the Bayes' theorem. The third equality multiplies both sides of the equation by $P(T = t|Z = z)$. The fourth equality arises from (63). The fifth equality is due the fact that T is deterministic conditioned on $\boldsymbol{S}$ and Z. Thus $P(T = t|\boldsymbol{S} = \boldsymbol{s}, Z = z)$ is either zero or one. The sixth equality simply reorders the terms of the summation. The seventh equality is due to (65). The eight equality is due to (64).

A.2 Proof of Proposition P.2

We seek to obtain a choice rule the stems from WARP and the budget set relationships defined by (15). It is useful to define some basic nomenclature to proof the proposition.

If we fix the instrument to a value $z \in supp(Z)$, then all the bundles $(t, g); g \in \mathcal{B}_i(z, t)$ for any $t \in supp(T)$ are said to be available for family i. If a family prefers a bundle (t, g) instead of (t', g') when both are available, then (t, g) is said to be directly and strictly revealed preferred to (t', g'), that is, $(t, g) \succ_i^d (t', g')$. In particular, if a family i chooses choice t when the IV value is fixed to z, that is, $T_i(z) = t$, then there exists a bundle (t, g^*) for some $g^* \in \mathcal{B}_i(z, t)$ that is strictly revealed preferred to all available bundles, namely, all the bundles $(t', g'); g' \in \mathcal{B}_i(z, t')$ for any choices t' that are different than t. Notationally, we have that $(t, g^*) \succ_i^d (t', g') forall g' \in \mathcal{B}_i(z, t'); t' \in supp(T)\backslash\{t\}$.

The WARP criteria of Richter (1971) states that if bundle (t, g) is directly and strictly revealed preferred to (t', g'), that is, $(t, g) \succ_i^d (t', g')$, then (t', g') cannot be revealed preferred to (t, g), namely, $(t, g) \succ_i^d (t', g') \Rightarrow (t', g') \not\succ_i^d (t, g)$.

We are now equipped to prove the proposition. Suppose that family i chooses choice t instead of t' when the IV value is fixed to z. Thus there exist a bundle (t, g^*) for some $g^* \in \mathcal{B}_i(z, t)$ such that $(t, g^*) \succ_i^d (t', g')$ for all $g' \in \mathcal{B}_i(z, t')$.

Now consider a shift of the IV from z to z'. Suppose that $0 \leq \boldsymbol{L}[z', t] - \boldsymbol{L}[z, t]$ holds. According to (15), the budget set $\mathcal{B}_i(z', t)$ is at least as big as $\mathcal{B}_i(z, t')$. In particular, the bundle (t, g^*) is available. Moreover, suppose that $0 \leq \boldsymbol{L}[z', t] - \boldsymbol{L}[z, t]$ also holds. This mean that the budget set $\mathcal{B}_i(z', t')$ is not larger than $\mathcal{B}_i(z, t')$ and, according to WARP, the bundle (t, g^*) is preferred to all the bundles $(t', g'); g' \in \mathcal{B}_i(z', t')$. Thereby, family i will not choose any bundle $(t', g'); g' \in \mathcal{B}_i(z', t')$ over (t, g^*). Consequently, we have that $T_i(z')$ cannot be t', namely, $t_i(z') \neq t'$.

A.3 WARP P.2 Subsumes Standard Monotonicity Conditions (10)–(12)

This section shows that Proposition **P.2** is able to subsume and outperform the conditions (10)–(12). The monotonicity condition (10), states that $\mathbf{1}[T_i(z_c) = t_l] \leq \mathbf{1}[T_i(z_e) = t_l]$. It comprise two choice restrictions: $T_i(z_c) = t_l \Rightarrow T_i(z_e) \neq t_h$ and $T_i(z_c) = t_l \Rightarrow T_i(z_e) \neq t_m$. These restrictions can be by applying **P.2** to t_l against t_h, t_m when the IV changes from z_c to z_e :

$$T_i(z_c) = t_l \text{ and } \boldsymbol{L}[z_e, t_h] - \boldsymbol{L}[z_c, t_h] = 0 \leq 1 \leq 1 = \boldsymbol{L}[z_e, t_l] - \boldsymbol{L}[z_c, t_l] \Rightarrow T_i(z_e) \neq t_h.$$
$$T_i(z_c) = t_l \text{ and } \boldsymbol{L}[z_e, t_m] - \boldsymbol{L}[z_c, t_m] = 0 \leq 1 \leq 1 = \boldsymbol{L}[z_e, t_l] - \boldsymbol{L}[z_c, t_l] \Rightarrow T_i(z_e) \neq t_m.$$

The monotonicity condition (11), states that $\mathbf{1}[T_i(z_c) \in \{t_m, t_l\}] \leq \mathbf{1}[T_i(z_8) \in \{t_m, t_l\}]$. It comprise two choice restrictions: $T_i(z_c) = t_l \Rightarrow T_i(z_8) \neq t_h$ and $T_i(z_c) = t_m \Rightarrow T_i(z_8) \neq t_h$. These restrictions can be by applying **P.2** to t_l, t_m against t_h when the IV changes from z_c to z_8 :

$$T_i(z_c) = t_l \text{ and } \boldsymbol{L}[z_8, t_h] - \boldsymbol{L}[z_c, t_h] = 0 \leq 0 \leq 1 = \boldsymbol{L}[z_8, t_l] - \boldsymbol{L}[z_c, t_l] \Rightarrow T_i(z_8) \neq t_h.$$
$$T_i(z_c) = t_m \text{ and } \boldsymbol{L}[z_8, t_h] - \boldsymbol{L}[z_c, t_h] = 0 \leq 0 \leq 1 = \boldsymbol{L}[z_8, t_m] - \boldsymbol{L}[z_c, t_m] \Rightarrow T_i(z_8) \neq t_h.$$

The monotonicity condition (12), states that $\mathbf{1}[T_i(z_e) = t_m] \leq \mathbf{1}[T_i(z_8) = t_m]$. It comprise two choice restrictions: $T_i(z_e) = t_m \Rightarrow T_i(z_8) \neq t_h$ and $T_i(z_e) = t_m \Rightarrow T_i(z_8) \neq t_l$. These restrictions can be by applying **P.2** to t_m against t_h, t_e when the IV changes from z_e to z_8 :

$$T_i(z_e) = t_m \text{ and } \boldsymbol{L}[z_8, t_h] - \boldsymbol{L}[z_e, t_h] = 0 \leq 0 \leq 1 = \boldsymbol{L}[z_8, t_m] - \boldsymbol{L}[z_e, t_m] \Rightarrow T_i(z_e) \neq t_h.$$
$$T_i(z_e) = t_m \text{ and } \boldsymbol{L}[z_8, t_l] - \boldsymbol{L}[z_e, t_l] = 0 \leq 0 \leq 1 = \boldsymbol{L}[z_8, t_m] - \boldsymbol{L}[z_e, t_m] \Rightarrow T_i(z_e) \neq t_m.$$

Proposition **P.2** yields additional choice restrictions that are not subsumed by the monotonicity conditions (10)–(12). For example, equation (66) applies **P.2** to t_m against t_h when the IV changes from z_e to z_c :

$$T_i(z_e) = t_m \text{ and } \boldsymbol{L}[z_c, t_h] - \boldsymbol{L}[z_c, t_h] = 0 \leq 0 = \boldsymbol{L}[z_c, t_m] - \boldsymbol{L}[z_e, t_m] \Rightarrow T_i(z_c) \neq t_h. \quad (66)$$

Equation (66) generates the choice restriction $T_i(z_e) = t_m \Rightarrow T_i(z_c) \neq t_h$. This restriction is not implied by the monotonicity conditions (10)–(12). Nevertheless, the choice restriction is intuitive. Note that neither z_e or z_c offers incentives towards choice t_h or t_m. Thus, if a family chooses t_m under z_e, then the family has no incentives to switch its decision towards t_h under z_c.

A.4 Proof of Proposition P.3

The incentive requirement in **P.2** compares two choices and two instrumental variables, namely,

$$\boldsymbol{L}[z', t'] - \boldsymbol{L}[z, t'] \leq 0 \leq \boldsymbol{L}[z', t] - \boldsymbol{L}[z, t]. \quad (67)$$

There are six possibilities for distinct choices (t, t') such that $t \in \{t_h, t_m, t_l\}$ and $t' \in \{t_h, t_m, t_l\} \setminus \{t\}$. There are also six possibilities for the set of distinct instrumental values (z, z') such that $z \in \{z_c, z_8, z_e\}$ and $z' \in \{z_c, z_8, z_e\} \setminus \{z\}$. Thus, there are a total of 36 choice requirements of the type in (67) that can be checked using MTO data. Only 20 of these 36 possibilities are biding. The resulting 20 choice restrictions are presented in Table A.1. These restrictions are summarized into the eight choice restrictions displayed in Table A.2. The two last choice restrictions of Table A.2 are redundant given the first six restrictions.

In total, the revealed preference analysis generates seven choice restrictions. The seventh choice restriction is due to the Normal choice assumption (27), that is, $T_i(z_c) \neq t_h \Rightarrow T_i(z_8) = T_i(z_c)$.

Table A.1: Choice Restrictions Due to WARP

#	Revealed Choice	Incentive Inequalities					Choice Statement
	$T_i(z) = t$	$\boldsymbol{L}[z',t'] - \boldsymbol{L}[z,t'] \leq 0 \leq \boldsymbol{L}[z',t] - \boldsymbol{L}[z,t]$					$T(z') \neq t'$
1	$T_i(z_c) = t_h,$	$\boldsymbol{L}[z_e,t_m] - \boldsymbol{L}[z_c,t_m] = 0$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_e,t_h] - \boldsymbol{L}[z_c,t_h]$	$T_i(z_e) \neq t_m$
2	$T_i(z_c) = t_m,$	$\boldsymbol{L}[z_8,t_h] - \boldsymbol{L}[z_c,t_h] = 0$	$\leq$	0	$\leq$	$1 = \boldsymbol{L}[z_8,t_m] - \boldsymbol{L}[z_c,t_m]$	$T_i(z_8) \neq t_h$
3	$T_i(z_c) = t_m,$	$\boldsymbol{L}[z_e,t_h] - \boldsymbol{L}[z_c,t_h] = 0$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_e,t_m] - \boldsymbol{L}[z_c,t_m]$	$T_i(z_e) \neq t_h$
4	$T_i(z_c) = t_l,$	$\boldsymbol{L}[z_8,t_h] - \boldsymbol{L}[z_c,t_h] = 0$	$\leq$	0	$\leq$	$1 = \boldsymbol{L}[z_8,t_l] - \boldsymbol{L}[z_c,t_l]$	$T_i(z_8) \neq t_h$
5	$T_i(z_c) = t_l,$	$\boldsymbol{L}[z_e,t_h] - \boldsymbol{L}[z_c,t_h] = 0$	$\leq$	0	$\leq$	$1 = \boldsymbol{L}[z_e,t_l] - \boldsymbol{L}[z_c,t_l]$	$T_i(z_e) \neq t_h$
6	$T_i(z_c) = t_l,$	$\boldsymbol{L}[z_e,t_m] - \boldsymbol{L}[z_c,t_m] = 0$	$\leq$	0	$\leq$	$1 = \boldsymbol{L}[z_e,t_l] - \boldsymbol{L}[z_c,t_l]$	$T_i(z_e) \neq t_m$
7	$T_i(z_8) = t_h,$	$\boldsymbol{L}[z_c,t_m] - \boldsymbol{L}[z_8,t_m] = -1$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_c,t_h] - \boldsymbol{L}[z_8,t_h]$	$T_i(z_c) \neq t_m$
8	$T_i(z_8) = t_h,$	$\boldsymbol{L}[z_e,t_m] - \boldsymbol{L}[z_8,t_m] = -1$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_e,t_h] - \boldsymbol{L}[z_8,t_h]$	$T_i(z_e) \neq t_m$
9	$T_i(z_8) = t_h,$	$\boldsymbol{L}[z_c,t_l] - \boldsymbol{L}[z_8,t_l] = -1$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_c,t_h] - \boldsymbol{L}[z_8,t_h]$	$T_i(z_c) \neq t_l$
10	$T_i(z_8) = t_h,$	$\boldsymbol{L}[z_e,t_l] - \boldsymbol{L}[z_8,t_l] = 0$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_e,t_h] - \boldsymbol{L}[z_8,t_h]$	$T_i(z_e) \neq t_l$
11	$T_i(z_8) = t_l,$	$\boldsymbol{L}[z_e,t_h] - \boldsymbol{L}[z_8,t_h] = 0$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_e,t_l] - \boldsymbol{L}[z_8,t_l]$	$T_i(z_e) \neq t_h$
12	$T_i(z_8) = t_l,$	$\boldsymbol{L}[z_e,t_m] - \boldsymbol{L}[z_8,t_m] = -1$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_e,t_l] - \boldsymbol{L}[z_8,t_l]$	$T_i(z_e) \neq t_m$
13	$T_i(z_e) = t_h,$	$\boldsymbol{L}[z_c,t_m] - \boldsymbol{L}[z_e,t_m] = 0$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_c,t_h] - \boldsymbol{L}[z_e,t_h]$	$T_i(z_c) \neq t_m$
14	$T_i(z_e) = t_h,$	$\boldsymbol{L}[z_c,t_l] - \boldsymbol{L}[z_e,t_l] = -1$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_c,t_h] - \boldsymbol{L}[z_e,t_h]$	$T_i(z_c) \neq t_l$
15	$T_i(z_e) = t_h,$	$\boldsymbol{L}[z_8,t_l] - \boldsymbol{L}[z_e,t_l] = 0$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_8,t_h] - \boldsymbol{L}[z_e,t_h]$	$T_i(z_8) \neq t_l$
16	$T_i(z_e) = t_m,$	$\boldsymbol{L}[z_c,t_h] - \boldsymbol{L}[z_e,t_h] = 0$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_c,t_m] - \boldsymbol{L}[z_e,t_m]$	$T_i(z_c) \neq t_h$
17	$T_i(z_e) = t_m,$	$\boldsymbol{L}[z_8,t_h] - \boldsymbol{L}[z_e,t_h] = 0$	$\leq$	0	$\leq$	$1 = \boldsymbol{L}[z_8,t_m] - \boldsymbol{L}[z_e,t_m]$	$T_i(z_8) \neq t_h$
18	$T_i(z_e) = t_m,$	$\boldsymbol{L}[z_c,t_l] - \boldsymbol{L}[z_e,t_l] = -1$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_c,t_m] - \boldsymbol{L}[z_e,t_m]$	$T_i(z_c) \neq t_l$
19	$T_i(z_e) = t_m,$	$\boldsymbol{L}[z_8,t_l] - \boldsymbol{L}[z_e,t_l] = 0$	$\leq$	0	$\leq$	$1 = \boldsymbol{L}[z_8,t_m] - \boldsymbol{L}[z_e,t_m]$	$T_i(z_8) \neq t_l$
20	$T_i(z_e) = t_l,$	$\boldsymbol{L}[z_8,t_h] - \boldsymbol{L}[z_e,t_h] = 0$	$\leq$	0	$\leq$	$0 = \boldsymbol{L}[z_8,t_l] - \boldsymbol{L}[z_e,t_l]$	$T_i(z_8) \neq t_h$

This table displays the binding choice restrictions generated by the WARP restriction below

$$\text{If} \quad T_i(z) = t \quad \text{and} \quad \boldsymbol{L}[z',t'] - \boldsymbol{L}[z,t'] \leq 0 \leq \boldsymbol{L}[z',t] - \boldsymbol{L}[z,t] \quad \text{then} \quad T_i(z') \neq t'.$$

when applied to the MTO incentive matrix:

$$\text{MTO Incentive Matrix } \boldsymbol{L} = \begin{array}{c} \begin{matrix} t_h & t_m & t_l \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{matrix} \\ z_c \\ z_8 \\ z_e \end{matrix}$$

Table A.2: Summary of Choice Restrictions generated by applying WARP to the MTO Incentive Matrix

#	Choice Restrictions		
4,5,6	$T_i(z_c) = t_l$	$\Rightarrow$	$T_i(z_e) = t_l$ and $T_i(z_8) \neq t_h$
2,3	$T_i(z_c) = t_m$	$\Rightarrow$	$T_i(z_e) \neq t_h$ and $T_i(z_8) \neq t_h$
16,17,18,19	$T_i(z_e) = t_m$	$\Rightarrow$	$T_i(z_c) = t_m$ and $T_i(z_8) = t_m$
13,14,15	$T_i(z_e) = t_h$	$\Rightarrow$	$T_i(z_c) = t_h$ and $T_i(z_8) \neq t_l$
7,8,9,10	$T_i(z_8) = t_h$	$\Rightarrow$	$T_i(z_c) = t_h$ and $T_i(z_e) = t_h$
11,12	$T_i(z_8) = t_l$	$\Rightarrow$	$T_i(z_e) = t_l$
1	$T_i(z_c) = t_l$	$\Rightarrow$	$T_i(z_e) \neq t_m$
20	$T_i(z_e) = t_l$	$\Rightarrow$	$T_i(z_8) \neq t_h$

A.5 Identification of Counterfactual Outcomes in T.1

Heckman and Pinto (2018) show that for any response matrix $\boldsymbol{R}$ and any subset of response-types $\mathcal{S} \subset supp(\boldsymbol{S})$, we have that:

$$E(Y(t)|\boldsymbol{S} \in \mathcal{S}) \quad \text{is identified if and only if} \quad \boldsymbol{b}(\mathcal{S})'(\boldsymbol{K}_t)\boldsymbol{b}(\mathcal{S}) = 0, \tag{68}$$

where:

1. $\boldsymbol{I}$ is the identity matrix,
2. $\boldsymbol{B}_t \equiv \mathbf{1}[\boldsymbol{R} = t]$ is a binary matrix that indicates which elements in $\boldsymbol{R}$ are equal to t,
3. $\boldsymbol{B}_t^+$ is the Moore-Penrose pseudo-inverse of $\boldsymbol{B}_t$,
4. $\boldsymbol{K}_t = (\boldsymbol{I}_{7\times 7} - \boldsymbol{B}_t^+\boldsymbol{B}_t)$ is a symmetric $N_S \times N_S$ matrix,
5. $\boldsymbol{b}(\mathcal{S})$ is the binary vector that indicates which response-type belongs to $\mathcal{S}$, namely:

$$\boldsymbol{b}(\mathcal{S}) = [\mathbf{1}[\boldsymbol{s}_{ah} \in \mathcal{S}], \mathbf{1}[\boldsymbol{s}_{am} \in \mathcal{S}], \mathbf{1}[\boldsymbol{s}_{al} \in \mathcal{S}], \mathbf{1}[\boldsymbol{s}_{fc} \in \mathcal{S}], \mathbf{1}[\boldsymbol{s}_{pl} \in \mathcal{S}], \mathbf{1}[\boldsymbol{s}_{pm} \in \mathcal{S}], \mathbf{1}[\boldsymbol{s}_{ph} \in \mathcal{S}]]'.$$

The Moore-Penrose matrix is unique and always exists for any real-valued matrix Magnus and Neudecker (1999). If $E(Y(t)|\boldsymbol{S} \in \mathcal{S})$ is identified, then it can be evaluated by the expression

$$E(Y(t)|\boldsymbol{S} \in \mathcal{S}) = \frac{\boldsymbol{b}(\mathcal{S})'\boldsymbol{B}_t^+ (\boldsymbol{Q}_Z(t) \odot \boldsymbol{P}_Z(t))}{\boldsymbol{b}(\mathcal{S})'\boldsymbol{B}_t^+ \boldsymbol{P}_Z(t)}.$$

Consider the following matrices for the choice of high-poverty neighborhood t_h :

$$\boldsymbol{B}_{t_h} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \boldsymbol{B}_{t_h}^+ = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & 0 & -0.5 \\ 0.5 & 0 & -0.5 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix} \Rightarrow K_{t_h} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & -0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{69}$$

Equation (71) presents the binary matrix $\boldsymbol{B}_{t_l} = \mathbf{1}[\boldsymbol{R} = t_l]$ which has the same dimension of the response matrix $\boldsymbol{R}$ in (32) and takes value 1 if the respective element in $\boldsymbol{R}$ is t_l or zero otherwise. The seven columns of $\boldsymbol{B}_{t_l}$ are associated with the respective sequence of response-types:

$$\boldsymbol{s}_{ah}, \boldsymbol{s}_{am}, \boldsymbol{s}_{al}, \boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}, \boldsymbol{s}_{pm}, \boldsymbol{s}_{ph}.$$

The equation also shows the pseudo-inverse of $\boldsymbol{B}_{t_l}$, that is $\boldsymbol{B}_{t_l}^{+}$ and the matrix $(\boldsymbol{I}_{7\times 7} - \boldsymbol{B}_{t_h}^{+}\boldsymbol{B}_{t_h})$. Theorem (**T.1**) states that the counterfactual mean $E(Y(t_h)|\boldsymbol{S} \in \mathcal{S})$ is identified for the following sets of response-types $\mathcal{S} \in \{\{\boldsymbol{s}_{ah}\}, \{\boldsymbol{s}_{ph}\}, \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}\}\}$. The indicator vectors for each subset of response-types are:

$$\begin{aligned} \boldsymbol{b}(\{\boldsymbol{s}_{ah}\}) &= [1,0,0,0,0,0,0]', \\ \boldsymbol{b}(\{\boldsymbol{s}_{ph}\}) &= [0,0,0,0,0,0,1]', \\ \boldsymbol{b}(\{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}\}) &= [0,0,0,1,1,0,0]'. \end{aligned}$$

Note that $\boldsymbol{K}_{t_h}$ is symmetric. Its first row/column are zero which implies that $\boldsymbol{b}(\{\boldsymbol{s}_{ah}\})'\boldsymbol{K}_{\boldsymbol{t_h}}\boldsymbol{b}(\{\boldsymbol{s}_{ah}\}) = 0$, thus by (68), $E(Y(t_h)|\boldsymbol{S} = \boldsymbol{s_{ah}})$ is identified. The last row/column of $\boldsymbol{K}_{t_h}$ are zero, which implies that $\boldsymbol{b}(\{\boldsymbol{s}_{ph}\})'\boldsymbol{K}_{\boldsymbol{t_h}}\boldsymbol{b}(\{\boldsymbol{s}_{ph}\}) = 0$, thus by (68), $E(Y(t_h)|\boldsymbol{S} = \boldsymbol{s_{ph}})$ is identified. Lastly, it is easy to see that the sum of the fourth and fifth rows/columns of $\boldsymbol{K}_{t_h}$ are zero, which implies that $\boldsymbol{b}(\{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}\})'\boldsymbol{K}_{\boldsymbol{t_h}}\boldsymbol{b}(\{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}\}) = 0$, and thereby $E(Y(t_h)|\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl})$ is identified.

The matrices for medium-poverty neighborhood t_m are displayed below:

$$\boldsymbol{B}_{t_m} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \boldsymbol{B}_{t_m}^{+} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ -0.5 & 0.5 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \\ -0.5 & 0.5 & 0 \end{bmatrix} \Rightarrow \boldsymbol{K}_{t_m} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5 & 0 & 0 & 0.5 \end{bmatrix} \tag{70}$$

Applying the same analysis of the choice t_h to choice t_m we have that:

$$\begin{aligned} \boldsymbol{b}(\{\boldsymbol{s}_{am}\}) = [0,1,0,0,0,0,0]' &\Rightarrow \boldsymbol{b}(\{\boldsymbol{s}_{am}\})'\boldsymbol{K}_{t_m}\boldsymbol{b}(\{\boldsymbol{s}_{am}\}) = 0 \\ \boldsymbol{b}(\{\boldsymbol{s}_{pm}\}) = [0,0,0,0,0,1,0]' &\Rightarrow \boldsymbol{b}(\{\boldsymbol{s}_{pm}\})'\boldsymbol{K}_{t_m}\boldsymbol{b}(\{\boldsymbol{s}_{pm}\}) = 0 \\ \boldsymbol{b}(\{\boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}\}) = [0,0,0,1,0,0,1]' &\Rightarrow \boldsymbol{b}(\{\boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}\})'\boldsymbol{K}_{t_m}\boldsymbol{b}(\{\boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}\}) = 0. \end{aligned}$$

According to equation (68), we have that $E(Y(t_m)|\boldsymbol{S} = \boldsymbol{s}_{am})$, $E(Y(t_m)|\boldsymbol{S} = \boldsymbol{s}_{pm})$ and $E(Y(t_m)|\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}\})$ are identified.

The matrices for low-poverty neighborhood t_l are displayed below:

$$\boldsymbol{B}_{t_l} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \Rightarrow \boldsymbol{B}_{t_l}^{+} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -0.5 & 0.5 \\ -1 & 1 & 0 \\ 0 & -0.5 & 0.5 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \boldsymbol{K}_{t_l} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{71}$$

Applying the same analysis of the choice t_h to choice t_m we have that:

$$\boldsymbol{b}(\{\boldsymbol{s}_{al}\}) = [0,0,1,0,0,0,0]' \Rightarrow \boldsymbol{b}(\{\boldsymbol{s}_{al}\})'\boldsymbol{K}_{t_l}\boldsymbol{b}(\{\boldsymbol{s}_{al}\}) = 0$$
$$\boldsymbol{b}(\{\boldsymbol{s}_{pl}\}) = [0,0,0,0,1,0,0]' \Rightarrow \boldsymbol{b}(\{\boldsymbol{s}_{pl}\})'\boldsymbol{K}_{t_l}\boldsymbol{b}(\{\boldsymbol{s}_{pl}\}) = 0$$
$$\boldsymbol{b}(\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pm}\}) = [0,0,0,1,0,1,0]' \Rightarrow \boldsymbol{b}(\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pm}\})'\boldsymbol{K}_{t_l}\boldsymbol{b}(\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pm}\}) = 0.$$

According to equation (68), we have that $E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{al}), E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{pl})$ and $E(Y(t_l)|\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pm}\})$ are identified.

A.6 Identification of Response-type Probabilities in T.2

The matrix version of equation (9) is given by:

$$\boldsymbol{P}_Z(t) = \boldsymbol{B}_t \cdot \boldsymbol{P}_S; \quad t \in \{t_h, t_m, t_l\}, \tag{72}$$

where $\boldsymbol{P}_Z(t) = [P(T=t|Z=z_c), P(T=t|Z=z_8), P(T=t|Z=z_e)]'$, is the vector of propensity scores, $\boldsymbol{P}_S = [P(\boldsymbol{S}=\boldsymbol{s}_{ah}), P(\boldsymbol{S}=\boldsymbol{s}_{am}), P(\boldsymbol{S}=\boldsymbol{s}_{al}), P(\boldsymbol{S}=\boldsymbol{s}_{fc}), P(\boldsymbol{S}=\boldsymbol{s}_{pl}), P(\boldsymbol{S}=\boldsymbol{s}_{pm}), P(\boldsymbol{S}=\boldsymbol{s}_{ph})]'$, is the vector of response-type probabilities, and $\boldsymbol{B}_t = \mathbf{1}[\boldsymbol{R}=t]$ is a binary 3×7 matrix that indicates which elements in $\boldsymbol{R}$ are equal to $t\in\{t_l,t_m,t_h\}$. We can stack equation (72) across neighborhood choices to generate the following equation:

$$\begin{bmatrix} \boldsymbol{P}_Z(t_h) \\ \boldsymbol{P}_Z(t_l) \\ \boldsymbol{P}_Z(t_m) \end{bmatrix} = \boldsymbol{B}_T \cdot \boldsymbol{P}_S, \text{ where } \boldsymbol{B}_T \equiv \begin{bmatrix} \boldsymbol{B}_{t_h} \\ \boldsymbol{B}_{t_m} \\ \boldsymbol{B}_{t_l} \end{bmatrix}. \tag{73}$$

Heckman and Pinto (2018) show that the response-type probabilities are point-identified if and only if the column rank of $\boldsymbol{B}_T$ is equal to the number of response-types, namely $\text{rank}(\boldsymbol{B}_T)=7$. The matrix $\boldsymbol{B}_T$ is presented below:

$$\boldsymbol{B}_T \equiv \begin{bmatrix} \boldsymbol{B}_{t_h} \\ \boldsymbol{B}_{t_m} \\ \boldsymbol{B}_{t_l} \end{bmatrix} = \begin{bmatrix} 1&0&0&1&1&0&1 \\ 1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&1 \\ 0&1&0&0&0&1&0 \\ 0&1&0&1&0&1&1 \\ 0&1&0&0&0&0&0 \\ 0&0&1&0&0&0&0 \\ 0&0&1&0&1&0&0 \\ 0&0&1&1&1&1&0 \end{bmatrix} \tag{74}$$

It is easy to check that the columns are linear independent, which implies that $\text{rank}(\boldsymbol{B}_T)=7$. The response-type probabilities are identified as $\boldsymbol{P}_S = \boldsymbol{B}_T^{+}[\boldsymbol{P}_Z(t_h)', \boldsymbol{P}_Z(t_l)', \boldsymbol{P}_Z(t_m)']'$, where $\boldsymbol{B}_T^{+}$ is the Moore-Penrose pseudo-inverse of $\boldsymbol{B}_T$. We can use the fact that $P(T=t_h|Z=z)+P(T=t_m|Z=z)+P(T=t_l|Z=z)=1$ for each $z\in\{z_c,z_8,z_e\}$ in order to write the vector of response-type probabilities as:

$$\begin{bmatrix} P(\boldsymbol{S}=\boldsymbol{s}_{ah}) \\ P(\boldsymbol{S}=\boldsymbol{s}_{am}) \\ P(\boldsymbol{S}=\boldsymbol{s}_{al}) \\ P(\boldsymbol{S}=\boldsymbol{s}_{fc}) \\ P(\boldsymbol{S}=\boldsymbol{s}_{pl}) \\ P(\boldsymbol{S}=\boldsymbol{s}_{pm}) \\ P(\boldsymbol{S}=\boldsymbol{s}_{ph}) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P(T=t_h|Z=z_c) \\ P(T=t_h|Z=z_8) \\ P(T=t_h|Z=z_e) \\ P(T=t_m|Z=z_c) \\ P(T=t_m|Z=z_8) \\ P(T=t_m|Z=z_e) \\ P(T=t_l|Z=z_c) \\ P(T=t_l|Z=z_8) \\ P(T=t_l|Z=z_e) \end{bmatrix}.$$

A.7 Proof of Proposition P.5

According to equation 6, the TOT parameter that compares the experimental and control groups is given by:

$$TOT_e = \frac{E(Y|Z=z_e) - E(Y|Z=z_c)}{P(C_e=1|Z=z_e)},$$

where C_e indicates if the experimental voucher is used. We can rewrite the terms in the in the numerator of TOT_e as:

$$E(Y|Z=z_e) = \sum_{t \in \{t_h, t_m, t_l\}} E(Y|T=t, Z=z_e) P(T=t|Z=z_e)$$

$$\text{and } E(Y|Z=z_e) = \sum_{t \in \{t_h, t_m, t_l\}} E(Y|T=t, Z=z_c) P(T=t|Z=z_c)$$

We can then use equation (8) and the response matrix (28) to express each term $E(Y|T=t, Z=z)P(T=t|Z=z)$ as a sum of counterfactual outcomes conditioned on response-types:

$$\begin{aligned} E(Y|Z=z_e) = & E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ah})P(\boldsymbol{S}=\boldsymbol{s}_{ah}) + E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{am})P(\boldsymbol{S}=\boldsymbol{s}_{am}) \\ & + E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{al})P(\boldsymbol{S}=\boldsymbol{s}_{al}) + E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{fc})P(\boldsymbol{S}=\boldsymbol{s}_{fc}) \\ & + E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{pl})P(\boldsymbol{S}=\boldsymbol{s}_{pl}) + E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{pm})P(\boldsymbol{S}=\boldsymbol{s}_{pm}) + E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ph})P(\boldsymbol{S}=\boldsymbol{s}_{ph}), \end{aligned}$$

$$\begin{aligned} E(Y|Z=z_c) = & E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ah})P(\boldsymbol{S}=\boldsymbol{s}_{ah}) + E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{am})P(\boldsymbol{S}=\boldsymbol{s}_{am}) \\ & + E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{al})P(\boldsymbol{S}=\boldsymbol{s}_{al}) + E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{fc})P(\boldsymbol{S}=\boldsymbol{s}_{fc}) \\ & + E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{pl})P(\boldsymbol{S}=\boldsymbol{s}_{pl}) + E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{pm})P(\boldsymbol{S}=\boldsymbol{s}_{pm}) + E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{ph})P(\boldsymbol{S}=\boldsymbol{s}_{ph}). \end{aligned}$$

The voucher effect $E(Y|Z=z_e) - E(Y|Z=z_c)$ can be rewritten as:

$$\begin{aligned} E(Y|Z=z_e) - E(Y|Z=z_c) = & E(Y(t_l) - Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{fc})P(\boldsymbol{S}=\boldsymbol{s}_{fc}) \\ & + E(Y(t_l) - Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{pl})P(\boldsymbol{S}=\boldsymbol{s}_{pl}) + E(Y(t_l) - Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{pm})P(\boldsymbol{S}=\boldsymbol{s}_{pm}), \\ = & E(Y(t_l) - Y(t_h)|\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}\})P(\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}\}) + E(Y(t_l) - Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{pm})P(\boldsymbol{S}=\boldsymbol{s}_{pm}). \end{aligned}$$

Thus TOT effect that compares experimental versus control groups is given by:

$$TOT_e = \frac{E(Y(t_l)-Y(t_h)|\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pl}\})P(\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pl}\}) + E(Y(t_l)-Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{pm})P(\boldsymbol{S}=\boldsymbol{s}_{pm})}{P(\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pl},\boldsymbol{s}_{pm}\})}\cdot\xi_e,$$
$$s.t.\xi_e = \frac{P(\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pl},\boldsymbol{s}_{pm}\})}{P(C_e=1|Z=z_e)}.$$

We can apply analogous arguments to examine the causal content of the the TOT parameter that compares the Section 8 and control groups. The TOT parameter is given by:

$$TOT_8 = \frac{E(Y|Z=z_8)-E(Y|Z=z_c)}{P(C_8=1|Z=z_8)},$$

where C_8 indicates if the Section 8 voucher is used. The expectation $E(Y|Z=z_8)$ can be expressed as:

$$\begin{aligned}E(Y|Z=z_8) = {} & E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ah})P(\boldsymbol{S}=\boldsymbol{s}_{ah}) + E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{am})P(\boldsymbol{S}=\boldsymbol{s}_{am})\\ & + E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{al})P(\boldsymbol{S}=\boldsymbol{s}_{al}) + E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{fc})P(\boldsymbol{S}=\boldsymbol{s}_{fc})\\ & + E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{pl})P(\boldsymbol{S}=\boldsymbol{s}_{pl}) + E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{pm})P(\boldsymbol{S}=\boldsymbol{s}_{pm}) + E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{ph})P(\boldsymbol{S}=\boldsymbol{s}_{ph}).\end{aligned}$$

The voucher effect $E(Y|Z=z_8)-E(Y|Z=z_c)$ can be rewritten as:

$$\begin{aligned}E(Y|Z=z_8)-E(Y|Z=z_c) = {} & E(Y(t_m)-Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{fc})P(\boldsymbol{S}=\boldsymbol{s}_{fc})\\ & + E(Y(t_l)-Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{pl})P(\boldsymbol{S}=\boldsymbol{s}_{pl}) + E(Y(t_m)-Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ph})P(\boldsymbol{S}=\boldsymbol{s}_{ph}),\\ = {} & E(Y(t_l)-Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{pl})P(\boldsymbol{S}=\boldsymbol{s}_{pl}) + E(Y(t_m)-Y(t_h)|\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{ph}\})P(\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{ph}\}).\end{aligned}$$

Thus TOT effect that compares Section 8 versus control groups is given by:

$$TOT_8 = \frac{E(Y(t_l)-Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{pl})P(\boldsymbol{S}=\boldsymbol{s}_{pl}) + E(Y(t_m)-Y(t_h)|\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{ph}\})P(\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{ph}\}}{P(\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pl},\boldsymbol{s}_{ph}\})}\cdot\xi_e,$$
$$s.t.\xi_e = \frac{P(\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pl},\boldsymbol{s}_{ph}\})}{P(C_8=1|Z=z_8)}.$$

A.8 Proof of Theorem T.3

Table A.3 presents which response-types are eliminated by the monotonicity conditions in **T.3**. The response-types that survive the elimination process are precisely those displayed in the response matrix 28. It remains to prove that no other set of monotonicity conditions of the type described by unordered monotonicity 44 generates the same response matrix. To do so, it suffices to show that a change the direction of each the monotonicity conditions violates a pattern of counterfactual choices displayed in the response matrix. For convenience, the response matrix is presented below.

$$\boldsymbol{R} = \begin{array}{c} \begin{matrix} \boldsymbol{s}_{ah} & \boldsymbol{s}_{am} & \boldsymbol{s}_{al} & \boldsymbol{s}_{fc} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{ph} \end{matrix} \\ \begin{bmatrix} t_h & t_m & t_l & t_h & t_h & t_m & t_h \\ t_h & t_m & t_l & t_m & t_l & t_m & t_m \\ t_h & t_m & t_l & t_l & t_l & t_l & t_h \end{bmatrix} \end{array} \quad \begin{matrix} T_i(z_c) \\ T_i(z_8) \\ T_i(z_e) \end{matrix}$$

The items below indicate which counterfactual choice pattern is violated if we reverse the direction of each of the monotonicity conditions in **T.3**.

1. $\mathbf{1}[T_i(z_c) = t_h] \leq \mathbf{1}[T_i(z_8) = t_h]$ violates the choice pattern in response-type $\boldsymbol{s}_{ph}$ when the instrument switches from z_c to z_8.

2. $\mathbf{1}[T_i(z_8) = t_h] \geq \mathbf{1}[T_i(z_e) = t_h]$ violates the choice pattern in response-type $\boldsymbol{s}_{ph}$ when the instrument switches from z_e to z_8.

3. $\mathbf{1}[T_i(z_e) = t_h] \geq \mathbf{1}[T_i(z_c) = t_h]$ violates the choice pattern in response-type $\boldsymbol{s}_{fc}$ when the instrument switches from z_c to z_e.

4. $\mathbf{1}[T_i(z_c) = t_m] \geq \mathbf{1}[T_i(z_8) = t_m]$ violates the choice pattern in response-type $\boldsymbol{s}_{fc}$ when the instrument switches from z_8 to z_c.

5. $\mathbf{1}[T_i(z_8) = t_m] \leq \mathbf{1}[T_i(z_e) = t_m]$ violates the choice pattern in response-type $\boldsymbol{s}_{pm}$ when the instrument switches from z_8 to z_e.

6. $\mathbf{1}[T_i(z_e) = t_m] \geq \mathbf{1}[T_i(z_c) = t_m]$ violates the choice pattern in response-type $\boldsymbol{s}_{pm}$ when the instrument switches from z_c to z_e.

7. $\mathbf{1}[T_i(z_c) = t_l] \geq \mathbf{1}[T_i(z_8) = t_l]$ violates the choice pattern in response-type $\boldsymbol{s}_{pl}$ when the instrument switches from z_8 to z_c.

8. $\mathbf{1}[T_i(z_8) = t_l] \geq \mathbf{1}[T_i(z_e) = t_l]$ violates the choice pattern in response-type $\boldsymbol{s}_{pm}$ when the instrument switches from z_e to z_8.

9. $\mathbf{1}[T_i(z_e) = t_l] \leq \mathbf{1}[T_i(z_c) = t_l]$ violates the choice pattern in response-type $\boldsymbol{s}_{pl}$ when the instrument switches from z_e to z_c.

A.9 Proof of Theorem T.4

Item (i) of the theorem states that the choice indicator $D_t = \mathbf{1}[T = t]$ can be expressed as the separable equation $D_t = \mathbf{1}[P_t(Z) \geq U_t]$ where U_t is an unobserved variables that is uniformity distributed in $[0,1]$. The proof consisting in constructing a variable U_t such that $D_t = \mathbf{1}[P_t(Z) \geq U_t]$ w.p.1. and to show that the constructed variable has uniform distribution.

Remark 1.1. The theorem describes the IV model using the Rubin-Holland causal model, which employs the language of potential outcomes in (1)–(3) to define IV model. The main advantage of using the Rubin-Holland causal model is its simplicity. However, this causal framework has a major drawback. The language of potential outcomes severely limits the interpretation of the IV model, and, in particular, the interpretation of variable U_t. Appendix D uses structural equation to equivalently describe the IV model. I refer to Appendix D for the interpretation of the unobserved variables in the IV model.

The first step to proving item (i) is to show the choice indicator D_t can be expressed as a threshold crossing indicator. This fact stems from the triangular property of the MTO response matrix, namely, each binary matrix $\boldsymbol{B}_t = \mathbf{1}[\boldsymbol{R} = t]; t \in \{t_h, t_m, t_l\}$ can be written as a lower triangular matrix as displayed in equations (35), (42), and (43) of Section 4.

Table A.3: Elimination of MTO Response-types by the Unordered Monotonicity Condition

Counterfactual Choices	All 27 Possible Response-types																										
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
$T_i(z_c)$	t_h	t_h	t_h	t_h	t_h	t_h	t_h	t_h	t_h	t_m	t_m	t_m	t_m	t_m	t_m	t_m	t_m	t_m	t_l	t_l	t_l	t_l	t_l	t_l	t_l	t_l	t_l
$T_i(z_8)$	t_h	t_h	t_h	t_m	t_m	t_m	t_l	t_l	t_l	t_h	t_h	t_h	t_m	t_m	t_m	t_l	t_l	t_l	t_h	t_h	t_h	t_m	t_m	t_m	t_l	t_l	t_l
$T_i(z_e)$	t_h	t_m	t_l	t_h	t_m	t_l	t_h	t_m	t_l	t_h	t_m	t_l	t_h	t_m	t_l	t_h	t_m	t_l	t_h	t_m	t_l	t_h	t_m	t_l	t_h	t_m	t_l
Monotonicity 1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓	✓	✓	✓	✓
Monotonicity 2	✓	✗	✗	✓	✓	✓	✓	✓	✓	✓	✗	✗	✓	✓	✓	✓	✓	✓	✓	✗	✗	✓	✓	✓	✓	✓	✓
Monotonicity 3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✓	✓	✗	✓	✓	✗	✓	✓	✗	✓	✓	✗	✓	✓	✗	✓	✓
Monotonicity 4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓	✓	✗	✗	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓
Monotonicity 5	✓	✗	✓	✓	✓	✓	✓	✗	✓	✓	✗	✓	✓	✓	✓	✓	✗	✓	✓	✗	✓	✓	✓	✓	✓	✗	✓
Monotonicity 6	✓	✗	✓	✓	✗	✓	✓	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✓	✓	✗	✓	✓	✗	✓
Monotonicity 7	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗	✗	✗	✓	✓	✓
Monotonicity 8	✓	✓	✓	✓	✓	✓	✗	✗	✓	✓	✓	✓	✓	✓	✓	✗	✗	✓	✓	✓	✓	✓	✓	✓	✗	✗	✓
Monotonicity 9	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✓	✗	✗	✓	✗	✗	✓
Not Eliminated	1			4		6			9					14	15												27

The top section of this table lists all the 27 possible response-types that the response variable $S_i = [T_i(z_c)\ ,\ T_i(z_8)\ ,\ T_i(z_e)]$ can take. Rows present the counterfactual neighborhood choices that would arise if a family were assigned to control group, Section 8 and experimental group, that is $T_i(z_c), T_i(z_8)$ and $T_i(z_e)$ respectively. Columns present all the values of response-type as choices range over $supp(T) = \{t_h, t_m, t_l\}$. The second section of this table indicate whether the response-type in the column of the first panel violates any of the following monotonicity relations:

	Z-pairs	Values of T	Unordered Monotonicity Relations		
Monotonicity Relation 1	(z_c, z_8)	t_h	$\mathbf{1}[T_i(z_c) = t_h]$	$\geq$	$\mathbf{1}[T_i(z_8) = t_h]$
Monotonicity Relation 2	(z_8, z_e)	t_h	$\mathbf{1}[T_i(z_8) = t_h]$	$\leq$	$\mathbf{1}[T_i(z_e) = t_h]$
Monotonicity Relation 3	(z_e, z_c)	t_h	$\mathbf{1}[T_i(z_c) = t_h]$	$\geq$	$\mathbf{1}[T_i(z_e) = t_h]$
Monotonicity Relation 4	(z_c, z_8)	t_m	$\mathbf{1}[T_i(z_c) = t_m]$	$\leq$	$\mathbf{1}[T_i(z_8) = t_m]$
Monotonicity Relation 5	(z_8, z_e)	t_m	$\mathbf{1}[T_i(z_8) = t_m]$	$\geq$	$\mathbf{1}[T_i(z_e) = t_m]$
Monotonicity Relation 6	(z_e, z_c)	t_m	$\mathbf{1}[T_i(z_e) = t_m]$	$\leq$	$\mathbf{1}[T_i(z_c) = t_m]$
Monotonicity Relation 7	(z_c, z_8)	t_l	$\mathbf{1}[T_i(z_c) = t_l]$	$\leq$	$\mathbf{1}[T_i(z_8) = t_l]$
Monotonicity Relation 8	(z_8, z_e)	t_l	$\mathbf{1}[T_i(z_8) = t_l]$	$\leq$	$\mathbf{1}[T_i(z_e) = t_l]$
Monotonicity Relation 9	(z_e, z_c)	t_l	$\mathbf{1}[T_i(z_e) = t_l]$	$\geq$	$\mathbf{1}[T_i(z_c) = t_l]$

A check mark sign indicates that the response-type indicated by the column in the top of the table does not violate the choice restriction indicated by the row. A cross sign indicates that the associated response-type violates the relation. The last row of the panel indicates the response-types that are not eliminated by any of the monotonicity relatinons.

Consider the case of low poverty neighborhoods t_l as our leading example. The triangular response matrix for t_l in (35) is displayed bellow for our convenience:

$$\boldsymbol{R}_l = \begin{array}{c} \begin{matrix} \boldsymbol{s}_{al} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{fc} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{ah} & \boldsymbol{s}_{am} & \boldsymbol{s}_{pl} \end{matrix} \\ \begin{bmatrix} t_l & t_h & t_h & t_m & t_h & t_m & t_h \\ t_l & t_l & t_m & t_m & t_h & t_m & t_m \\ t_l & t_l & t_l & t_l & t_h & t_m & t_h \end{bmatrix} \end{array} \begin{matrix} z_c \\ z_8 \\ z_e \end{matrix}$$

Let $\boldsymbol{B}_{t_l} \equiv \mathbf{1}[\boldsymbol{R}_l = t_l]$ be the binary matrix that takes 1 if the respective element in $\boldsymbol{R}_l$ is t_l, that is:

$$\boldsymbol{B}_{t_l} \equiv \mathbf{1}[\boldsymbol{R}_l = t_l] = \begin{array}{c} \begin{matrix} \boldsymbol{s}_{al} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{fc} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{ah} & \boldsymbol{s}_{am} & \boldsymbol{s}_{pl} \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} z_c \\ z_8 \\ z_e \end{matrix}$$

It is useful to relabel the indexes of the z-values according to increasing values of row-sums and relabel the indexes of the response-types according to decreasing values of the columns-sums:

$$\boldsymbol{B}_{t_l} \equiv \begin{array}{c} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} z_1 \\ z_2 \\ z_3 \end{matrix} \tag{75}$$

Proposition **P.1**, enable us to relate propensity scores and response-type probabilities by the following equation:

$$P(T = t | Z = z) = \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} \mathbf{1}[T = t | \boldsymbol{S} = \boldsymbol{s}, Z = z] \, P(\boldsymbol{S} = \boldsymbol{s}). \tag{76}$$

We can use the matrix version of equation (76) to relate propensity scores and response-type probabilities as following:

$$\begin{bmatrix} P_{t_l}(z_1) \\ P_{t_l}(z_2) \\ P_{t_l}(z_3) \end{bmatrix} = \underbrace{\begin{array}{c} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}}_{\text{Binary matrix } B_{t_l} = \mathbf{1}[\boldsymbol{R}_l = t_l]} \begin{bmatrix} P(\boldsymbol{S} = \boldsymbol{s}_1) \\ P(\boldsymbol{S} = \boldsymbol{s}_2) \\ P(\boldsymbol{S} = \boldsymbol{s}_3) \\ P(\boldsymbol{S} = \boldsymbol{s}_4) \\ P(\boldsymbol{S} = \boldsymbol{s}_5) \\ P(\boldsymbol{S} = \boldsymbol{s}_6) \\ P(\boldsymbol{S} = \boldsymbol{s}_7) \end{bmatrix}. \tag{77}$$

Equation (77) generates the following relationships between the relabeled propensity scores and response-type probabilities:

$$P_{t_l}(z_1) = \sum_{j=1}^{1} P(\boldsymbol{S} = \boldsymbol{s}_j) \tag{78}$$

$$P_{t_l}(z_2) = \sum_{j=1}^{2} P(\boldsymbol{S} = \boldsymbol{s}_j) \tag{79}$$

$$P_{t_l}(z_3) = \sum_{j=1}^{4} P(\boldsymbol{S} = \boldsymbol{s}_j) \tag{80}$$

The key property that arises from the triangular property of matrix (75) is that we can express each of the elements of B_{t_l} in (75) as an indicator of an inequality between the propensity score and the sum of the response-type probabilities. To see this property, let $\boldsymbol{B}_{t_l}[z_i, \boldsymbol{s}_j]; (i,j) \in \{1,2,3\} \times \{1,...,7\}$ denotes the elements of matrix $\boldsymbol{B}_{t_l}$. The matrix equation (77) renders the following properties of these elements:

$$\boldsymbol{B}_{t_l}[z_i, \boldsymbol{s}_j] = 1 \quad \Leftrightarrow \quad P_l(z_i) \geq \sum_{j'=1}^{j} P(\boldsymbol{S} = \boldsymbol{s}_{j'}), \tag{81}$$

$$\text{or} \quad \boldsymbol{B}_{t_l}[z_i, \boldsymbol{s}_j] = 0 \quad \Leftrightarrow \quad P_l(z_i) < \sum_{j'=1}^{j} P(\boldsymbol{S} = \boldsymbol{s}_{j'}). \tag{82}$$

Note that $\boldsymbol{B}_{t_l}[z_i, \boldsymbol{s}_j] = \mathbf{1}[T = t_l | Z = z_i, \boldsymbol{S} = \boldsymbol{s}_j]$, thus, we can express the choice indicator as:

$$\mathbf{1}[T = t_l | Z = z_i, \boldsymbol{S} = \boldsymbol{s}_j] = \mathbf{1}\left[P_l(z_i) \geq \sum_{j'=1}^{j} P(\boldsymbol{S} = \boldsymbol{s}_{j'})\right] \text{ for } (i,j) \in \{1,2,3\} \times \{1,...,7\}. \tag{83}$$

Equation 83 shows that the choice indicator (right-hand side) can be expressed as a the indicator of a separable inequality that compares the propensity score $P_l(z_i)$ with the sum of response-type probabilities.

There are several ways to construct a variable $U_{t_l} \sim Unif[0,1]$ such that $D_{t_l} = \mathbf{1}[P_t(Z) \geq U_{t_l}]$. For instance, let $U_1, ..., U_7$ be i.i.d. random variables uniformly distributed in $[0,1]$. Let U_{t_l} be defined as:

$$U_{t_l} = \sum_{j=1}^{7} \mathbf{1}[\boldsymbol{S} = \boldsymbol{s}_j] \cdot \left(\sum_{j'=0}^{j-1} P(\boldsymbol{S} = \boldsymbol{s}_{j-1}) + U_j \cdot P(\boldsymbol{S} = \boldsymbol{s}_j) \right), \text{ where } P(\boldsymbol{S} = \boldsymbol{s}_0) \equiv 0. \tag{84}$$

Variable U_{t_l} has a uniform distribution in $[\sum_{j'=0}^{j-1} P(\boldsymbol{S} = \boldsymbol{s}_{j'}), \sum_{j'=0}^{j} P(\boldsymbol{S} = \boldsymbol{s}_{j'})]$ conditional on $\boldsymbol{S} = \boldsymbol{s}_j$. Unconditionally, variable U_{t_l} has a uniform distribution in $[0,1]$. Let the indicator variable be defined as:

$$\tilde{D}_{t_l} = \mathbf{1}[P_{t_l}(Z) \geq U_{t_l}]. \tag{85}$$

Note that:

$$\left(\tilde{D}_{t_l} | Z = z_i, \boldsymbol{S} = \boldsymbol{s}_i\right) = \mathbf{1}[P_{t_l}(z_i) \geq U_{t_l,j}],$$
$$\text{where} \quad U_{t_l,j} \sim Unif\left[\sum_{j'=0}^{j-1} P(\boldsymbol{S} = \boldsymbol{s}_{j'}), \sum_{j'=0}^{j} P(\boldsymbol{S} = \boldsymbol{s}_{j'})\right].$$

According to (81)–(82), we have that $(\tilde{D}_{t_l} | Z = z_i, \boldsymbol{S} = \boldsymbol{s}_j) = \boldsymbol{B}_{t_l}[z_i, \boldsymbol{s}_j]$ for all (i,j) $\in \{1,2,3\} \times \{1,...,7\}$. Moreover, we have that $D_{t_l} \equiv \mathbf{1}[T = t_l]$, thus we can combine all results into:

$$(D_{t_l} | Z = z_i, \boldsymbol{S} = \boldsymbol{s}_j) = \mathbf{1}[T = t_l | Z = z_i, \boldsymbol{S} = \boldsymbol{s}_j] = \boldsymbol{B}_{t_l}[z_i, \boldsymbol{s}_j] = (\tilde{D}_{t_l} | Z = z_i, \boldsymbol{S} = \boldsymbol{s}_j).$$

In particular, we have that:

$$D_{t_l} = D_{t_l}(Z, \boldsymbol{S}) = (\tilde{D}_{t_l} | Z, \boldsymbol{S}) = \mathbf{1}[P_{t_l}(Z) \geq U_{t_l}].$$

As mentioned, the rationale for establishing that $D_{t_l} = \mathbf{1}[P_{t_l}(Z) \geq U_{t_l}]$ holds stems from the triangular property of the MTO matrix for choice t_l. The variables U_{t_h}, U_{t_m} can be constructed in the same fashion since the triangular property of the MTO matrix holds for t_h and t_m.

To prove the item (ii) of the theorem, first note that the exogeneity condition (2), $Z \perp\!\!\!\perp (Y(t), T(z))$, implies that $Z \perp\!\!\!\perp \boldsymbol{S}$, but U_t is a function of only $\boldsymbol{S}$, which implies that $Z \perp\!\!\!\perp (U_t, Y(t))$, and thereby $Z \perp\!\!\!\perp Y(t)|U_t$, holds. Thus, we have that:

$$\begin{aligned}
E(Y \cdot \mathbf{1}[T = t]|Z = z) &= E(Y(t) \cdot D_t|Z = z) && (86)\\
&= E(Y(t) \cdot \mathbf{1}[P_{t_l}(Z) \geq U_{t_l}]|Z = z) && (87)\\
&= E(Y(t) \cdot \mathbf{1}[P_{t_l}(z) \geq U_{t_l}]) && (88)\\
&= \int_0^{P_{t_l}(z)} E(Y(t)|U_{t_l} = u)du, && (89)
\end{aligned}$$

where the second equality uses $D_t = \mathbf{1}[P_t(Z) \geq U_{t_l}]$, the third equality is due to $Z \perp\!\!\!\perp (Y(t), U_t)$, and the fourth equality is due to $U_{t_l} \sim Unif[0, 1]$.

Let $z, z' \in supp(Z)$ such that $P_t(z') > P_t(z)$. Equation (89) enable us to write:

$$\begin{aligned}
E(YD_t|Z = z') - E(YD_t|Z = z) &= E(Y(t)\mathbf{1}[P_{t_l}(z') \geq U_{t_l}]) - E(Y\mathbf{1}[P_{t_l}(z) \geq U_{t_l}])\\
&= E(Y(t) \cdot (\mathbf{1}[P_{t_l}(z') \geq U_{t_l}] - \mathbf{1}[P_{t_l}(z) \geq U_{t_l}]))\\
&= E(Y(t) \cdot (\mathbf{1}[P_{t_l}(z') \geq U_{t_l} \geq P_{t_l}(z)]))\\
&= \int_{P_{t_l}(z)}^{P_{t_l}(z')} E(Y(t)|U_t = u)du
\end{aligned}$$

Therefore we have that:

$$\frac{E(YD_t|Z = z') - E(YD_t|Z = z)}{P_t(z') - P_t(z)} = \frac{\int_{P_t(z)}^{P_t(z')} E(Y(t)|U_t = u)du}{P_t(z') - P_t(z)}.$$

A.10 Proof of Theorem T.5

Assumptions (1)–(3) enable us to relate propensity scores and response-type probabilities by the following equation:

$$P_t(z) \equiv P(T = t|Z = z) = \sum_{\boldsymbol{s} \in supp(\boldsymbol{S})} \mathbf{1}[T = t|\boldsymbol{S} = \boldsymbol{s}, Z = z]\, P(\boldsymbol{S} = \boldsymbol{s}). \quad (90)$$

The equation above shows that each propensity score equals a sum of response-type probabilities. The triangular property of the MTO response matrix in (35), (42), and (43) enable us to map each propensity score to nested sets of response-types. In the case of t_l, we can use equation (90) and $\boldsymbol{R}_l$ in (35) to express the propensity scores as:

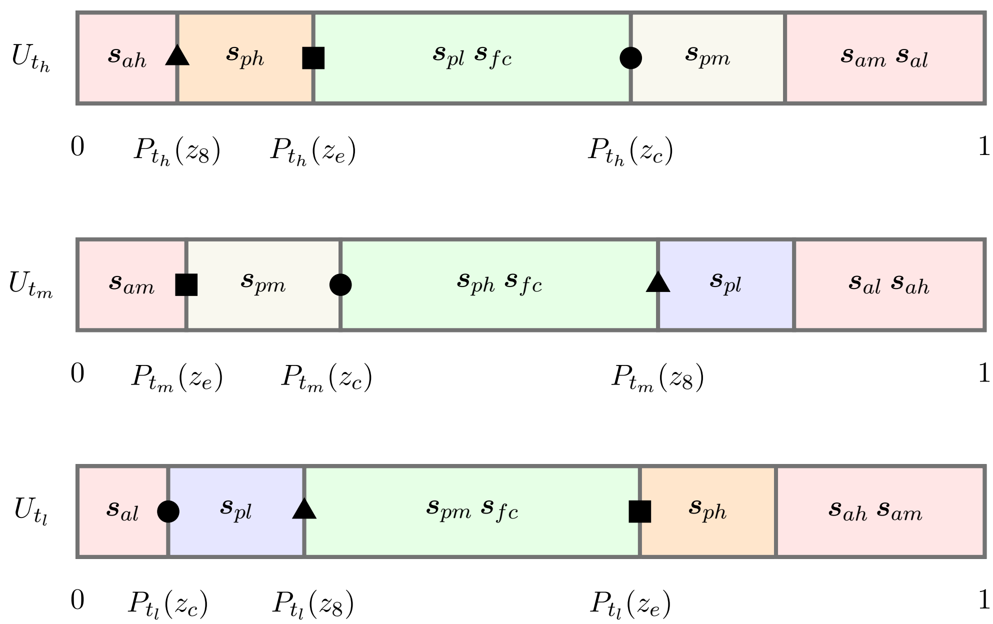
$$\begin{aligned}
P_{t_l}(z_c) &= \sum_{\boldsymbol{s} \in \{\boldsymbol{s}_{al}\}} P(\boldsymbol{S} = \boldsymbol{s}) \Rightarrow P_{t_l}(z_c) = P(\boldsymbol{S} \in \{\boldsymbol{s}_{al}\})\\
P_{t_l}(z_8) &= \sum_{\boldsymbol{s} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}\}} P(\boldsymbol{S} = \boldsymbol{s}) \Rightarrow P_{t_l}(z_8) = P(\boldsymbol{S} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}\})\\
P_{t_l}(z_e) &= \sum_{\boldsymbol{s} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}, \boldsymbol{s}_{fc}, \boldsymbol{s}_{pm}\}} P(\boldsymbol{S} = \boldsymbol{s}) \Rightarrow P_{t_l}(z_e) = P(\boldsymbol{S} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}, \boldsymbol{s}_{fc}, \boldsymbol{s}_{pm}\})
\end{aligned}$$

Note that the triangular property of the MTO response matrix maps the propensity scores into a family of nested sets of response-types. This sequence of nested sets determines the ordering of the response-types along the support of the variable U_t of the choice indicator $D_t = \mathbf{1}[P_t(Z) \geq U_t]$. Figure 2 displays the ordering of response-types for t_l. In summary, the sequence of the response-types associated with each variable $U_t; t \in \{t_h, t_m, , t_l\}$ is given below:

- The sequence of response-types associated with U_{t_h} is: $(\boldsymbol{s}_{ah}, \boldsymbol{s}_{ph}, \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}\}, \{\boldsymbol{s}_{pm}, \boldsymbol{s}_{am}, \boldsymbol{s}_{al}\})$
- The sequence of response-types associated with U_{t_m} is: $(\boldsymbol{s}_{am}, \boldsymbol{s}_{pm}, \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}\}, \{\boldsymbol{s}_{pl}, \boldsymbol{s}_{al}, \boldsymbol{s}_{ah}\})$
- The sequence of response-types associated with U_{t_l} is: $(\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}, \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pm}\}, \{\boldsymbol{s}_{ph}, \boldsymbol{s}_{ah}, \boldsymbol{s}_{am}\})$

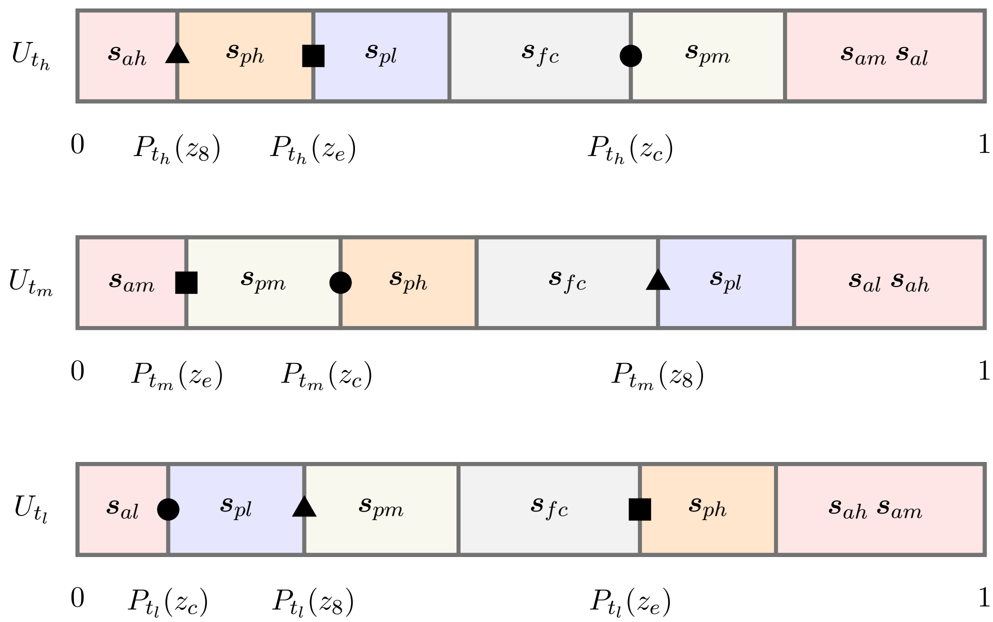
Figure A.1 presents a diagram that displays the ordering of the response-types associated with each variable U_t.

Figure A.1: Order of Response-types for Each Neighborhood Choice due to MTO Response Matrix



We seek to split three intervals: (1) $\{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}\}$ associated with U_{t_h}; (2) $\{\boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}\}$ associated with U_{t_m}; (3) $\{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pm}\}$ associated with U_{t_l}. The theorem considers the ordering of the response-types in Figure A.1. It investigates the case where the partial compliers $\boldsymbol{s}_{pl}, \boldsymbol{s}_{ph}, \boldsymbol{s}_{pm}$ precede the full-compliers $\boldsymbol{s}_{fc}$. This ordering scheme is displayed in the diagram of Figure A.2.

Figure A.2: Order of Response-types Assuming that Partial-compliers precede Full-compliers



It is useful to clarify the choice scheme displayed by the diagram of Figure A.2.

Consider the top bar which refers to variable U_{t_h}. The sequence of response-types displayed in the bar is $\boldsymbol{s}_{ah}, \boldsymbol{s}_{ph}, \boldsymbol{s}_{pl}, \boldsymbol{s}_{fc}, \boldsymbol{s}_{pm}, \{\boldsymbol{s}_{am}, \boldsymbol{s}_{al}\}$. According to the choice equation $D_{t_h} = \mathbf{1}[p_{t_h} \geq U_{t_h}]$, this sequence of response-types implies a specific choice scheme. If the propensity score of choice t_h is set to $p_{t_h} = P(\boldsymbol{S} = \boldsymbol{s}_{al})$, then, $\boldsymbol{s}_{ah}$-families choose t_h while the other family types will not. If the propensity score of choice t_h is set to $p_{t_h} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{ph}\})$, then, families of type $\boldsymbol{s}_{ah}$ and $\boldsymbol{s}_{ph}$ choose t_h while the other family types will not. This pattern continues according to the sequence of the response-types displayed for t_h. The symmetric choice procedure also holds for the remaining choices.

We seek to investigate the choice scheme generated by assuming that partial-compliers precede the full-compliers. In the case of t_h, it means that $\boldsymbol{s}_{pl}$ precedes $\boldsymbol{s}_{fc}$. Thus considering setting the propensity score of choice t_h to $p_{t_h} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{ah}, \boldsymbol{s}_{ph}, \boldsymbol{s}_{pl}\})$. This means that families of type $\boldsymbol{s}_{ah}, \boldsymbol{s}_{ph}$ and $\boldsymbol{s}_{pl}$ choose t_h, while the families associated with the remaining response-types, that is, $\boldsymbol{s}_{fc}, \boldsymbol{s}_{pm}, \boldsymbol{s}_{am}$ and $\boldsymbol{s}_{al}$ do not choose t_h. In particular, the full-compliers $\boldsymbol{s}_{fc}$ must choose t_m or t_l. These two possibilities are considered below:

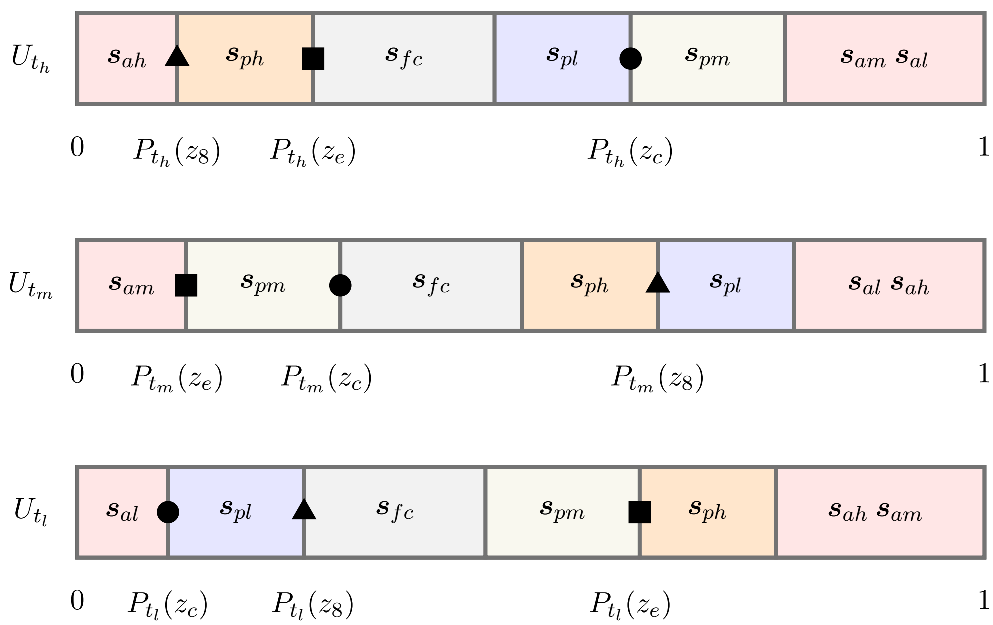
- Suppose that full-compliers $\boldsymbol{s}_{fc}$ choose t_m. Thus, according to the sequence of response-types of U_m (second bar), it must be the case that $p_m \geq P(\boldsymbol{S} \in \{\boldsymbol{s}_{am}, \boldsymbol{s}_{pm}, \boldsymbol{s}_{ph}, \boldsymbol{s}_{fc}\})$. Note that, according to equation $D_{t_h} = \mathbf{1}[p_{t_h} \geq U_{t_h}]$, it implies that families of type $\boldsymbol{s}_{am}, \boldsymbol{s}_{pm}$ and $\boldsymbol{s}_{ph}$ also choose t_m. There lies a contradiction since we have established that families of type $\boldsymbol{s}_{ph}$ were already choosing t_h.

- Suppose that full-compliers $\boldsymbol{s}_{fc}$ choose t_l. Thus, according to the sequence of response-types of U_l (third bar), it must be the case that $p_l \geq P(\boldsymbol{S} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}, \boldsymbol{s}_{pm}, \boldsymbol{s}_{fc}\})$. Thus, according to equation $D_{t_l} = \mathbf{1}[p_{t_l} \geq U_{t_l}]$, families of type $\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}$ and $\boldsymbol{s}_{pm}$ also choose t_l. There lies another contradiction since we have established that families of type $\boldsymbol{s}_{pl}$ were already choosing t_h.

In summary, for the full-compliers to choose t_m or t_l, it must be the case that either $\boldsymbol{s}_{pl}$ or $\boldsymbol{s}_{ph}$ do not exist. That is to say that either $P(\boldsymbol{S} = \boldsymbol{s}_{pl}) - 0$ or $P(\boldsymbol{S} = \boldsymbol{s}_{ph}) - 0$. The same rationale applies to the analysis of choices t_m and t_l. Namely, if we set $p_{t_m} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{am}, \boldsymbol{s}_{pm}, \boldsymbol{s}_{ph}\})$, then the full-compliers $\boldsymbol{s}_{fc}$ must choose either t_h or t_l, and this choice behavior is possible only if $P(\boldsymbol{S} = \boldsymbol{s}_{pm}) - 0$ or $P(\boldsymbol{S} = \boldsymbol{s}_{ph}) - 0$. Finally, if we set $p_{t_l} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}, \boldsymbol{s}_{pm}\})$, then the full-compliers $\boldsymbol{s}_{cf}$ must choose either t_h or t_m, and this choice behavior is possible only if $P(\boldsymbol{S} = \boldsymbol{s}_{pl}) - 0$ or $P(\boldsymbol{S} = \boldsymbol{s}_{pm}) - 0$.

The central message conveyed by this theorem is that assuming that partial-compliers precede full-compliers generates a contradictory choice scheme. Fortunately, these contradictions do not occur when full-compliers precede the partial-compliers. Figure A.3 presents the sequence of response-types in which the full-compliers precede the partial-compliers.

Figure A.3: Order of Response-types Assuming that Full-compliers precede Partial-compliers



Consider the sequence of response-types of U_{t_h} displayed in the first bar of Figure A.3. Setting the propensity score of choice t_h to $p_{t_h} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{ah}, \boldsymbol{s}_{ph}, \boldsymbol{s}_{fc}\})$. This means that families of type $\boldsymbol{s}_{al}, \boldsymbol{s}_{ph}$ and $\boldsymbol{s}_{fc}$ choose t_h. The families associated with the remaining response-types, that is, $\boldsymbol{s}_{pl}, \boldsymbol{s}_{pm}, \boldsymbol{s}_{am}$ and $\boldsymbol{s}_{al}$ do not choose t_h. This choice behavior is consistent with setting the propensity score of choice t_m to $p_{t_m} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{am}, \boldsymbol{s}_{pm}\})$, and the propensity score of choice t_l to $p_{t_l} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}\})$.

In the case of choice t_m, the sequence of response-types in Figure A.3 also provides consistent choice behaviors when we set the propensity scores to $p_{t_m} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{am}, \boldsymbol{s}_{pm}, \boldsymbol{s}_{fc}\})$, $p_{t_h} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{ah}, \boldsymbol{s}_{ph}\})$, and $p_{t_l} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}\})$.

In the case of choice t_l, the sequence of response-types in Figure A.3 also provides consistent choice behaviors when we set the propensity scores to $p_{t_l} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{al}, \boldsymbol{s}_{pl}, \boldsymbol{s}_{fc}\})$, $p_{t_m} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{am}, \boldsymbol{s}_{pm}\})$, and $p_{t_h} = P(\boldsymbol{S} \in \{\boldsymbol{s}_{ah}, \boldsymbol{s}_{ph}\})$.

B Defining Neighborhood Choices

The neighborhood choices are defined according to the eligibility criteria of MTO vouchers:

- Low poverty neighborhood (t_l) are the neighborhoods whose poverty level is below 10% according to the 1990 U.S. Census.
- High poverty neighborhood (t_h) are the housing projects targeted by the MTO experiment.
- Medium poverty neighborhood (t_m) are the remaining neighborhoods.

Each choice refers to the neighborhood decision at the beginning of the intervention. Thus, each neighborhood choice indicates the initial family decision of neighborhood relocation but also eventual subsequent moves made by the family.

Families using the vouchers were supposed to move from housing projects within six months of the voucher assignment. However, this rule was not strictly enforced: 17% of the families that used the Section 8 voucher and 36% of families that used the experimental voucher took more than 6 months to move. Thus the neighborhood choice depends on the voucher utilization, the neighborhood poverty level and also on the time that the family took to relocate.

It is useful to classify the families into three groups: stayers, compliers, and self-movers. *Stayers* are families that had not moved from their original housing projects since the intervention onset until the time of the interim evaluation in 2002. *Compliers* are families that use the experimental or Section 8 vouchers to relocate. *Self-movers* are families that had moved at the time of the interim evaluation without using the voucher. Table A.4 presents the distribution of these family types across sites. Around 20% of families that receive vouchers and 30% of the control families stayed in their original dwellings by the time of the interim evaluation. Self-movers totals 36% of experimental families and 24% of Section 8 families in 2002.

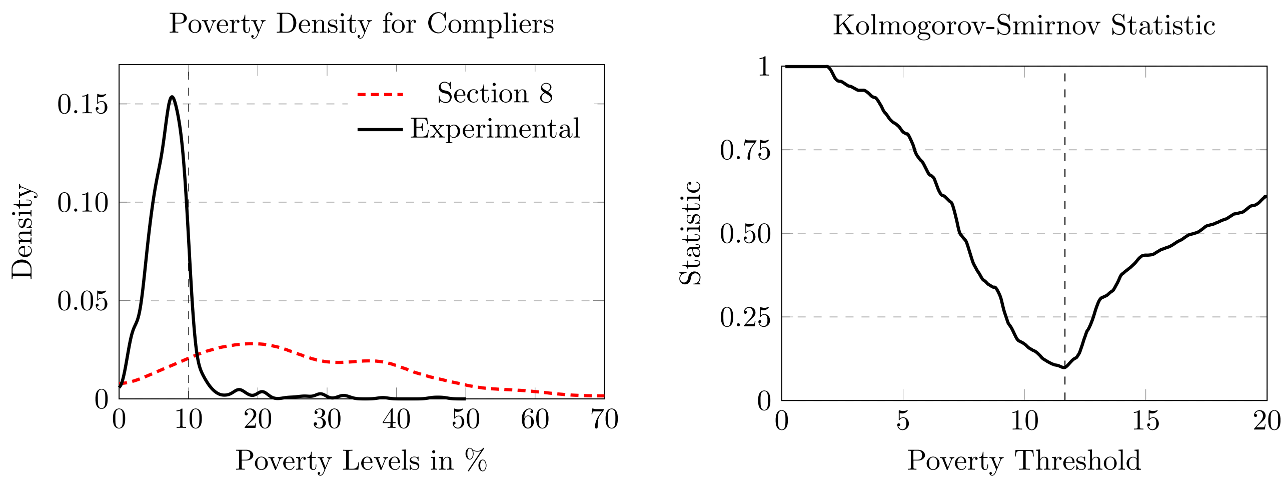
Table A.4: Relocation Rates by Site at the Time of the Interim Evaluation in 2002

Voucher Assignment	All Sites N	All Sites %	Relocation Decision	All Sites N	All Sites %	Baltimore N	Baltimore %	Boston N	Boston %	Chicago N	Chicago %	Los Angeles N	Los Angeles %	New York N	New York %
			Compliers	818	47%	146	58%	168	46%	155	34%	167	67%	182	45%
Experimental	1729	41%	Self-movers	618	36%	97	38%	149	41%	234	51%	53	21%	85	21%
			Stayers	293	17%	9	4%	49	13%	71	15%	30	12%	134	33%
			Compliers	716	59%	135	72%	129	48%	134	66%	130	77%	188	49%
Section 8	1209	28%	Self-movers	276	23%	45	24%	86	32%	55	27%	25	15%	65	17%
			Stayers	217	18%	7	4%	52	19%	13	6%	13	8%	132	34%
Control	1310	31%	Self-movers	917	70%	174	88%	240	74%	189	81%	172	66%	142	48%
			Stayers	393	30%	23	12%	86	26%	43	19%	88	34%	153	52%
Total	4248														

This tables describe the relocation of families by voucher assignment and site in 2002. MTO families are classified into three groups: (1) compliers – families that used the vouchers to relocate; (2) self-movers – families that had moved without the voucher at the time of the interim evaluation in 2002; (3) stayers – families that had not moved since intervention onset in 1994–1998 until the interim evaluation in 2002.

The neighborhood choices of stayers and compliers are easily characterized. The neighborhood choice of families who stay in their original dwellings is t_h. The experimental voucher can only be used to relocate to low poverty neighborhoods. Thus the neighborhood choice of experimental

Figure A.4: Poverty Densities and Threshold Investigation



The first graph presents the poverty density of chosen neighborhoods for families who comply with the Experimental and Section 8 vouchers. The second graph presents the Kolmogorov-Smirnov statistics (y-axis) between the poverty distribution of experimental compliers and the poverty distribution of Section 8 compliers that is right-bounded by a threshold (x-axis).

families that use the voucher is t_l. Families that decide to use the Section 8 voucher choose between low (t_l) or medium-poverty (t_m) neighborhoods. This ambiguity is resolved by assessing the poverty levels of the chosen neighborhoods.

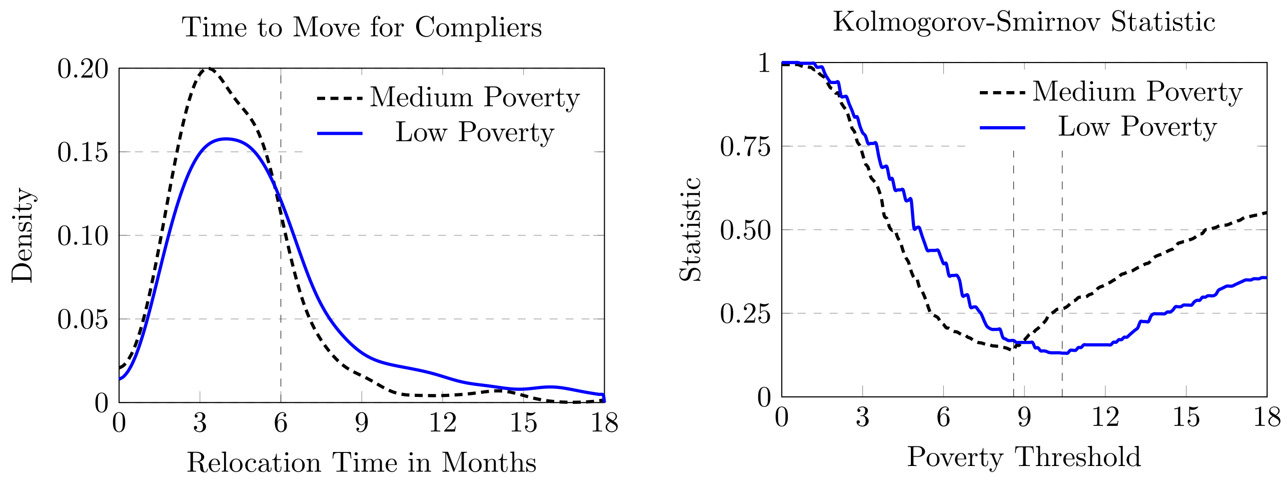
The experimental voucher defines low-poverty neighborhoods as those whose poverty level is below a soft target of 10%.[38] In practice, 11% of neighborhoods classified as low-poverty were slightly above the nominal threshold (first graph of Figure A.4). I employ a simple approach to address for this fact. I use the poverty distribution of Section 8 compliers to estimate a threshold that best conforms with the poverty distribution of low-poverty neighborhoods. Specifically, I estimate the threshold that minimizes the Kolmogorov-Smirnov statistic between the poverty distribution of Section 8 compliers and the poverty distribution of experimental compliers. The empirical threshold is 11.67% (second graph of Figure A.4).

It remains to determine the neighborhood choice for the self-movers, which comprise all families that have relocated between surveys. The goal is to identify families who decided to move by the time of the onset of the intervention. To do so, I explore the available information on the time spell from voucher assignment until the first relocation. I account for this fact using the same procedure that yields the poverty threshold. I estimate the threshold the minimizes the difference on the distribution on relocation time between compliers and self-movers. The first graph of Figure A.5 presents the distribution of relocation time for compliers while the second graph presents the Kolmogorov-Smirnov statistics for the difference on relocation time between compliers and self-movers. The corrected thresholds for relocation time are 8.6 months for medium-poverty neighborhoods and 10.6 months for low-poverty neighborhoods. The neighborhood choice of self-movers that relocate before these thresholds is set at either low or medium-poverty neighborhoods.

Figure A.6 summarizes the neighborhood decision of the MTO families by voucher assignment. Nearly 85% the control families choose high-poverty neighborhoods, 10% choose medium poverty neighborhoods and 3% choose low-poverty neighborhoods. Families that do not use the

[38] Using the 1990 US Census.

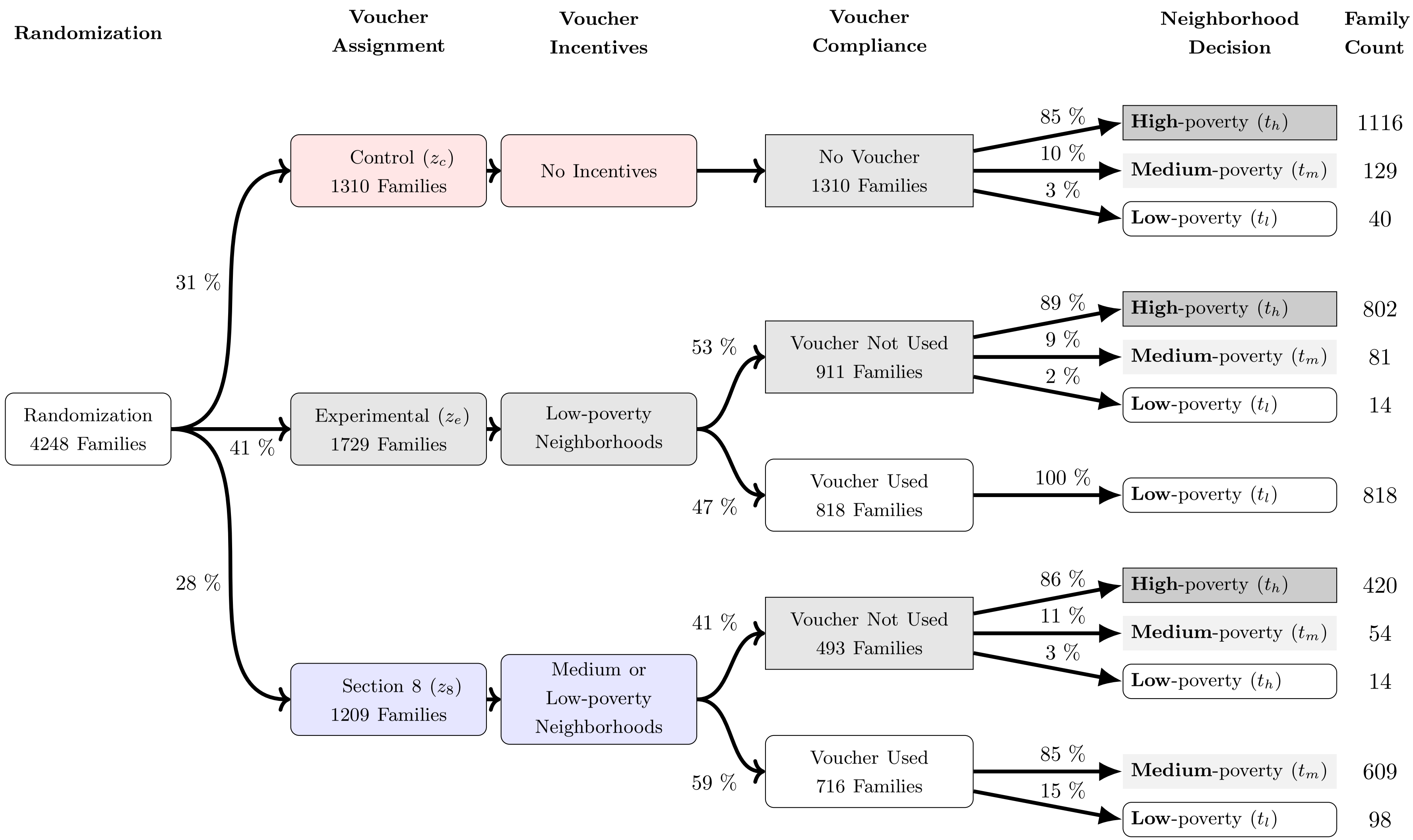
Figure A.5: Time to Relocate Densities and Threshold Investigation



The first graph presents the density of the time to relocate into low and medium poverty neighborhoods since voucher assignment for families who comply with the vouchers. Density estimates use the Gaussian Kernel with optimal bandwidth. The second graph presents the Kolmogorov-Smirnov statistics (y-axis) between the distribution time to relocate of voucher compliers and the distribution of time to relocate for self-movers that are right-bounded by a threshold (x-axis).

voucher share a similar composition of neighborhood choices. Around 15% of families that use the Section 8 voucher decide for low-poverty neighborhoods while 85% of Section 8 compliers choose medium-poverty neighborhoods. Neighborhood choices are robust across variations of the assignment procedure. For instance, we can generate alternative values for the neighborhood choices by setting the poverty threshold to its nominal value of 10% and the relocation time to 6 months generates. These values agree with the neighborhood choices described by the procedure above in 97% of the cases.

Figure A.6: **Neighborhood Relocation by Voucher Assignment and Compliance**



This figure describes the possible decision patterns of families in MTO that resulted from voucher assignment and family compliance. Low-poverty (t_l) neighborhoods are defined as those whose share of poor residents is below 10% according to the 1990 sensus (Orr et al., 2003). High poverty (t_h) neighborhoods are the housing projects originally targeted by the intervention. Medium poverty (t_m) neighborhoods are neither the high poverty nor the ones classified as low poverty. Families who stay in their baseline housing live in high poverty neighborhoods (t_h). Families who use the experimental voucher (z_e) relocate to low poverty neighborhoods (t_l). Families who use Section 8 voucher (z_8) can decide between low (t_l) or medium (t_m) poverty neighborhoods. Control families (z_c) and families that do not use the vouchers may choose freely among all three neighborhoods: high (t_h), medium (t_m) or low (t_l).

C TOT parameter of a Simplified Intervention

Consider a simpler intervention than MTO, which randomly assigns families living in high poverty neighborhoods to either a control group z_c or an experimental group z_c. The experimental group receives a voucher that incentivizes families to move to low poverty neighborhoods, while the control group receives no incentives. Families decide between two choices: remain in a high poverty neighborhood t_h or move to a low poverty neighborhood t_l.

Suppose that the investigators can prevent families from moving. Families assigned to the control group remain in the high poverty area. Families assigned to the experimental voucher that decide to not use the voucher do not move either. The only families that relocate are those assigned to the experimental voucher that agree to use the voucher.

This model admits two latent family types. *Compliers* are families who intend to use the experimental voucher in case they are assigned to it. *Non-compliers* are families that do not intend to use the experimental voucher. Notationally, $T_i(z) \in \{t_h, t_l\}$ denotes the potential choice of family i that is assigned to the group $z \in \{z_c, z_e\}$ and $Y_i(t)$ denotes the potential outcome of family i when the neighborhood choice is fixed at $t \in \{t_h, t_l\}$.

The response vector $\boldsymbol{S}_i = [T_i(z_c), T_i(z_e)]'$ lists the potential choices of a family i if it were assigned to the control and experimental groups respectively. If family i is a complier, then $\boldsymbol{S}_i = [t_h, t_l]'$, otherwise family i is a non-complier and its response vector is given by $\boldsymbol{S}_i = [t_h, t_h]'$. For notational simplicity, let $\boldsymbol{s}_n = [t_h, t_h]'$ denote the potential choices for non-compliers and let $\boldsymbol{s}_c = [t_h, t_l]'$ denote the potential choices for compliers.

IV assumptions apply, namely, $Z \perp\!\!\!\perp (T(z), Y(t))$ for all $(z,t) \in \{t_h, t_l\} \times \{z_c, z_e\}$. IV assumptions imply that $Z \perp\!\!\!\perp \boldsymbol{S}$. Otherwise stated, the IV randomization ensures that the share of family types in the experimental and control groups is the same. Consequently, the voucher take-up rate identifies the share of compliers, that is $P(\boldsymbol{S} = \boldsymbol{s}_c)$.

The intention-to-treat (ITT) parameter is given by $ITT = E(Y|Z = z_e) - E(Y|Z = z_c)$. It identifies the causal effect of being offered the experimental voucher. It is useful to express the ITT parameter as the weighted average of the voucher effects for compliers and non-compliers multiplied by their respective shares:

$$ITT = ITT_e(\boldsymbol{s}_n)P(\boldsymbol{S} = \boldsymbol{s}_{nc}) + ITT(\boldsymbol{s}_c)P(\boldsymbol{S} = \boldsymbol{s}_c), \tag{91}$$

$$ITT(\boldsymbol{s}_n) \equiv E(Y|Z = z_e, \boldsymbol{S} = \boldsymbol{s}_n) - E(Y|Z = z_c, \boldsymbol{S} = \boldsymbol{s}_n) = E(Y(t_h) - Y(t_h)|\boldsymbol{S} = \boldsymbol{s}_n) = 0, \tag{92}$$

$$ITT(\boldsymbol{s}_c) \equiv E(Y|Z = z_e, \boldsymbol{S} = \boldsymbol{s}_c) - E(Y|Z = z_c, \boldsymbol{S} = \boldsymbol{s}_c) = E(Y(t_l) - Y(t_h)|\boldsymbol{S} = \boldsymbol{s}_c), \tag{93}$$

where $ITT(\boldsymbol{s}_c), ITT(\boldsymbol{s}_n)$ denotes the the ITT parameter for families of type $\boldsymbol{s}_c$ and $\boldsymbol{s}_n$ respectively: The voucher effect for non-compliers is given by $ITT(\boldsymbol{s}_n)$ in (92), and it is zero as these families do not relocate. The voucher effect for compliers is given by $ITT(\boldsymbol{s}_c)$ in (93). Note that the compliers always choose t_h if assigned to z_c and choose t_l if assigned to z_e. Thus $ITT(\boldsymbol{s}_c)$ gives the causal effect of low versus high poverty neighborhoods on the outcome.

TOT in (6) is the ITT effect divided by the voucher take-up rate. In this setup, the TOT identifies the causal effect of the low versus high poverty neighborhoods for compliers:

$$TOT = \frac{ITT}{P(\boldsymbol{S} = \boldsymbol{s}_c)} \tag{94}$$

$$= \frac{ITT(\boldsymbol{s}_c)P(\boldsymbol{S} = \boldsymbol{s}_c)}{P(\boldsymbol{S} = \boldsymbol{s}_c)} \tag{95}$$

$$= E(Y(t_l) - Y(t_h)|\boldsymbol{S} = \boldsymbol{s}_c), \tag{96}$$

where the first equality is the definition of TOT, the second equality is due to (91)–(92). The third equation is due to (93).

D How IV Controls for Unobserved Characteristics

The main paper uses the language of potential outcomes to examine the identification in MTO. The primary advantage of the potential outcomes framework is its simplicity. The framework does not employ structural equations nor explicitly display unobserved variables. Its simplicity comes at a cost. It harms the interpretation of the causal model that generates the data. In particular, it is difficult to understand that the identification of causal parameters hinges on controlling for family unobserved characteristics. This section describes the causal model of MTO using structural equations. It clarifies the causal concepts underlying causality and the identification of causal parameters. See **?** for a recent discussion on causality, structural models and the limitations of the potential outcome framework.

The observed variables in MTO are: (1) voucher assignment $Z \in \{z_c, z_8, z_e\}$; (2) neighborhood choice $T \in \{t_h, t_m, t_l\}$; (3) outcome $Y \in \mathbb{R}$; and (4) baseline characteristics $\boldsymbol{X} \in \mathbb{R}^{|\boldsymbol{x}|}$. The MTO model is characterized by the following system of causal relations:

$$\text{Choice Equation}: \quad T = f_T(Z, \boldsymbol{V}, \boldsymbol{X}), \tag{97}$$

$$\text{Outcome Equation}: \quad Y = f_Y(T, \boldsymbol{V}, \boldsymbol{X}, \epsilon), \tag{98}$$

$$\text{Conditional Independence}: \quad Z \perp\!\!\!\perp \boldsymbol{V} | \boldsymbol{X}, \tag{99}$$

where $\boldsymbol{V}$ denotes the vector of family unobserved characteristics and ϵ is an unobserved variable satisfying $(Z, T, \boldsymbol{X}, \boldsymbol{V}) \perp\!\!\!\perp \epsilon$.[39] $\boldsymbol{V}$ is a confounding random vector that generates selection bias by causing both choice T and the outcome Y. Baseline variables $\boldsymbol{X}$ are family observed characteristics that cause T and Y. The experiment generates two required properties for Z to be an instrument: (98) implies that Z only affects Y through its impact on T (exclusion restriction); and (99) implies that Z is statistically independent of unobserved characteristics $\boldsymbol{V}$ given baseline variables $\boldsymbol{X}$.

The potential (counterfactual) outcome of family $i \in \mathcal{I}$ placed in neighborhood t is given by $Y_i(t) \equiv f_Y(t, \boldsymbol{V}_i, \boldsymbol{X}_i, \epsilon_i)$. It is the hypothetical outcome that would occur if the neighborhood choice of family i were exogenously set to $t \in \{t_h, t_m, t_l\}$. The potential choice of family i is given by $T_i(z) \equiv f_T(z, \boldsymbol{V}_i, \boldsymbol{X}_i)$. It is the choice that family i would take if it were exogenously assigned to voucher $z \in \{z_c, z_8, z_e\}$. Conditional independence (99) implies the IV exogeneity condition $\big(Y(t), T(z)\big) \perp\!\!\!\perp Z | \boldsymbol{X}$ for $(z, t) \in \{z_c, z_8, z_e\} \times \{t_h, t_m, t_l\}$. Outcome Y and choice T can be written in terms of potential variables as:

$$Y = \sum_{t \in \{t_l, t_m, t_h\}} D_t \cdot Y(t) = Y(T), \quad \text{and} \quad T = \sum_{z \in \{z_c, z_8, z_e\}} D_z \cdot T(z) = T(Z), \tag{100}$$

where $D_t = \mathbf{1}[T = t]; t \in \{t_h, t_m, t_l\}$ indicates neighborhood choices, $D_z = \mathbf{1}[Z = z]; z \in \{z_c, z_8, z_e\}$ indicates voucher assignment and $\mathbf{1}[A]$ is the indicator function that takes value 1 if event A is true and zero otherwise.

The causal effect of living in a low versus high-poverty neighborhood for family i is defined

[39] Measurement error and misspecification are possible sources of the unobserved error term ϵ.

as $Y_i(t_l) - Y_i(t_h)$. It is the difference in the potential outcome of family i if it were to reside in each of these two neighborhood types. If responses are heterogeneous, this individual effect is not identified since we only observe the potential outcome corresponding to the neighborhood chosen by the family. A mean neighborhood treatment effect is the expectation of individual effects, such as $Y_i(t_l) - Y_i(t_h)$, for subsets of families $i \in \mathcal{I}$. To gain intuition, it is useful to write the observed outcome of families $i \in \mathcal{I}$ that choose t_l or t_h as:

$$Y_i = \beta_0 + \beta_i D_{t_l,i} + \epsilon_i, \tag{101}$$

where $\beta_i = Y_i(t_l) - Y_i(t_h),\ \beta_0 = E(Y(t_h)),\ \epsilon_i = Y_i(t_h) - E(Y(t_h))$, and $D_{t,i} \equiv \mathbf{1}[T_i = t]$ is the choice indicator for a family i such that $T_i \in \{t_l, t_h\}$.

Equation (101) is a random coefficient model where β_i varies across $i \in \mathcal{I}$.[40] If $Y(t)$ and T were statistically independent, $Y(t) \perp\!\!\!\perp T$, then we could evaluate the average neighborhood effect $E(Y(t_h) - Y(t_h))$ by least squares taking mean differences. Selection bias induces a correlation between $Y(t)$ and T via $\boldsymbol{V}$. As a consequence, the regressor $D_{t_l,i}$ in (101) correlates with both the error term $\epsilon_i = Y_i(t_h) - E(Y(t_h))$ and the random coefficient $\beta_i = Y_i(t_l) - Y_i(t_h)$. Without further assumptions, neither least squares nor two-stage least squares identifies $E(Y(t_h) - Y(t_h))$.[41]

A popular identification strategy invokes a *matching* condition which assumes that T and $Y(t)$ are independent conditioned on $\boldsymbol{X}$, $Y(t) \perp\!\!\!\perp T|\boldsymbol{X}$. This assumption enables the analyst to identify counterfactual outcomes by controlling for $\boldsymbol{X} : E(Y|T = t, \boldsymbol{X}) = E(Y(t)|T = t, \boldsymbol{X}) = E(Y(t)|\boldsymbol{X})$, where the first equality is due to (100) and the second is due to $Y(t) \perp\!\!\!\perp T|\boldsymbol{X}$. The average neighborhood effect across all families in $i \in \mathcal{I}$ is obtained by integrating out $\boldsymbol{X}$:

$$E(Y(t_l) - Y(t_h)) = \int \left(E(Y|T = t_l, \boldsymbol{X} = \boldsymbol{x}) - E(Y|T = t_h, \boldsymbol{X} = \boldsymbol{x})\right) dF_{\boldsymbol{X}}(\boldsymbol{x}), \tag{102}$$

where $F_{\boldsymbol{X}}(\cdot)$ is the cumulative distribution function (CDF) of $\boldsymbol{X}$.

A matching assumption is not valid if there is selection bias on unobservables that are not in $\boldsymbol{X}$. However, it is always true that $Y(t) \perp\!\!\!\perp T|(\boldsymbol{X}, \boldsymbol{V})$ holds. The identification of causal effects hinges on controlling for $\boldsymbol{X}$ as well as for the unobservables $\boldsymbol{V}$. This paper presents a nonparametric method to control for $\boldsymbol{V}$. I suppress $\boldsymbol{X}$ henceforward to simplify notation. The analysis should be understood as conditioned on $\boldsymbol{X}$.

One identification strategy invokes a parametric model that uses Z to control for $\boldsymbol{V}$. Examples of such approach in the MTO literature are Aliprantis and Richter (2020); Chesher et al. (2020); Galiani et al. (2015). This paper takes a different approach. I exploit the instrument Z and the incentives in MTO to *nonparametrically* control for $\boldsymbol{V}$. The approach does not rely on any functional form assumptions, nor does it require intensive computational effort.

It is possible to control for $\boldsymbol{V}$ by partitioning families based on choice behavior described by *response-types* or *principal strata*, namely, the counterfactual choices that the family would take across the instrumental values.[42]

Let the *Response vector* $\boldsymbol{S}_i = [T_i(z_c), T_i(z_8), T_i(z_e)]'$ be the neighborhood choices made by family i when assigned to each of the instrumental values z_c, z_8, z_e. A response-type consists of a vector of

[40] Also called Switching Regression Model (Quandt, 1958, 1972).

[41] Later in this paper I present assumptions that enable the analyst to use a modified version of the two-stage least square regression to identify counterfactual outcomes.

[42] The use of response-types dates back to Balke and Pearl (1994) and Frangakis and Rubin (2002). See Pinto (2016) or Heckman and Pinto (2018) for a discussion.

choice values that $\boldsymbol{S}$ may take. For instance, family i that has *response-type* $\boldsymbol{S}_i = [t_h, t_m, t_l]'$ chooses a high-poverty neighborhood when offered z_c ($T_i(z_c) = t_h$), a medium-poverty neighborhood when offered z_8 ($T_i(z_8) = t_m$), and a low-poverty neighborhood when offered z_e ($T_i(z_e) = t_l$).

Choice T is determined by Z and $\boldsymbol{S}$. Given a response-type, choice T depends only on assignment Z, which is independent of its potential outcome $Y(t)$. Therefore $Y(t) \perp\!\!\!\perp T|\boldsymbol{S}$ holds. Intuitively, the neighborhood choice within a group of families that share the same response-type can be understood as if it were generated by randomized controlled trial RCT where Z determines the neighborhood assignment. If we knew all the families $i \in \mathcal{I}$ that have type $\boldsymbol{S}_i = [t_h, t_m, t_l]$, we would be able to identify the causal effect of low t_l versus high t_h from:

$$E(Y|Z = z_e, \boldsymbol{S} = [t_h, t_m, t_l]') - E(Y|Z = z_c, \boldsymbol{S} = [t_h, t_m, t_l]') \tag{103}$$
$$= E(Y|T = t_l, \boldsymbol{S} = [t_h, t_m, t_l]') - E(Y|T = t_h, \boldsymbol{S} = [t_h, t_m, t_l]'), \text{ due to response-type} \tag{104}$$
$$= E(Y(t_l)|T = t_l, \boldsymbol{S} = [t_h, t_m, t_l]') - E(Y(t_h)|T = t_h, \boldsymbol{S} = [t_h, t_m, t_l]'), \quad \text{due to (100)} \tag{105}$$
$$= E(Y(t_l) - Y(t_h)|\boldsymbol{S} = [t_h, t_m, t_l]), \qquad \text{due to } Y(t) \perp\!\!\!\perp T|\boldsymbol{S} \tag{106}$$

Response-types control for unobserved characteristics $\boldsymbol{V}$ by generating a useful partition of its support. Holding $\boldsymbol{X}$ fixed, the potential choice $\boldsymbol{T}(z) = f_T(z, \boldsymbol{V})$ depends only on $\boldsymbol{V}$. The set of unobserved characteristics corresponding to response-type $\boldsymbol{s} = [t_h, t_m, t_l]$ is given by:

$$\mathcal{V}_{\boldsymbol{s}} = \{\boldsymbol{v} \in supp(\boldsymbol{V}) \text{ such that } f_T(z_c, \boldsymbol{v}) = t_h, f_T(z_8, \boldsymbol{v}) = t_m, f_T(z_e, \boldsymbol{v}) = t_l\}.$$

Events $\boldsymbol{S} = \boldsymbol{s}$ and $\boldsymbol{V} \in \mathcal{V}_{\boldsymbol{s}}$ are equivalent. $Y(t) \perp\!\!\!\perp T|(\boldsymbol{S} = \boldsymbol{s})$ implies that $Y(t) \perp\!\!\!\perp T|(\boldsymbol{V} \in \mathcal{V}_{\boldsymbol{s}})$ holds. Conditioning on $\boldsymbol{S} = \boldsymbol{s}$ is equivalent to conditioning on the set of unobserved variables $\boldsymbol{V} \in \mathcal{V}_{\boldsymbol{s}}$ that renders the choice T statistically independent of the counterfactual outcomes $Y(t)$.[43] As $\boldsymbol{s}$ ranges in $supp(\boldsymbol{S})$, it spans the support of $\boldsymbol{V}$ as $supp(\boldsymbol{V}) = \bigcup_{\boldsymbol{s} \in supp(S)} \mathcal{V}_{\boldsymbol{s}}$.

Response-types are not observed, but we can express observed outcomes as a mixture of potential outcomes conditioned on response-types as written in equation (8) of the main paper.

E The Three-choice Model with a Parallel Design

Kirkeboen, Leuven, and Mogstad (2016), KLM henceforward, considers a three-valued treatment, $T \in \{t_0, t_1, t_2\}$, and a three-valued instrument $Z \in \{z_0, z_1, z_2\}$ where z_1 incentivizes t_1, z_2 incentivizes t_2, and z_0 is the baseline IV-value that offers no incentives. The model is consistent with a three-arm randomized trial design in which z_1, z_2 correspond to the intended treatment t_1, t_2 and z_0 stands for the control group. This is experiment is an example of the so called parallel design in the literature of randomized controlled trials. Choice incentives are characterized by the following incentive matrix:

$$\boldsymbol{L} = \begin{array}{c} \begin{matrix} t_0 & t_1 & t_2 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{matrix} z_0 \\ z_1 \\ z_2 \end{matrix} \tag{107}$$

[43] $\boldsymbol{S}$ plays the role of a control function in Heckman and Robb (1985) and Powell (1994) as well as an unobserved balancing score in Rosenbaum and Rubin (1983).

The incentive matrix (107) differs from the MTO incentive matrix (13) as z_1 incentivizes a single choice t_1, while the respective IV value in MTO, z_8, incentivizes two choices: t_m and t_l. MTO incentives justify tree monotonicity conditions (10)–(12), the incentive matrix (107) justifies only two:

$$\mathbf{1}[T_i(z_0) = t_1] \leq \mathbf{1}[T_i(z_1) = t_1] \tag{108}$$
$$\mathbf{1}[T_i(z_0) = t_2] \leq \mathbf{1}[T_i(z_2) = t_2]. \tag{109}$$

Monotonicity condition (108) states that a change in the instrument from z_0 to z_1 induces agents to shift their choice towards t_1 while (109) states that a change from z_0 to z_2 induces agents towards t_2. Panel B of Table (A.5) shows that monotonicity conditions (108)–(109) eliminate 12 out of the 27 possible response-types.

The remaining 15 response-types are displayed in response matrix (118):

$$\boldsymbol{R} = \begin{array}{c} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 & \boldsymbol{s}_9 & \boldsymbol{s}_{10} & \boldsymbol{s}_{11} & \boldsymbol{s}_{12} & \boldsymbol{s}_{13} & \boldsymbol{s}_{14} & \boldsymbol{s}_{15} \end{matrix} \\ \begin{bmatrix} t_0 & t_0 & t_0 & t_0 & t_0 & t_0 & t_0 & t_0 & t_0 & t_2 & t_1 & t_1 & t_2 & t_2 & t_2 \\ t_0 & t_0 & t_0 & t_1 & t_1 & t_1 & t_2 & t_2 & t_2 & t_2 & t_1 & t_1 & t_0 & t_1 & t_2 \\ t_0 & t_1 & t_2 & t_0 & t_1 & t_2 & t_0 & t_1 & t_2 & t_0 & t_1 & t_2 & t_2 & t_2 & t_2 \end{bmatrix} \end{array} \begin{matrix} T_i(z_0) \\ T_i(z_1) \\ T_i(z_2) \end{matrix} \tag{110}$$

The elimination of monotonicity conditions (108)–(109) are not sufficient to point-identify counterfactual outcomes and response-type probabilities.

The response matrix (118) enable us to express voucher effects as a weighted average of treatment effects conditioned on response-types. Voucher effect $E(Y|Z = z_1) - E(Y|Z = z_0)$ compares the second and first rows of the response matrix. We can use equation (8) to express this voucher effect as:

$$E(Y|Z = z_1) - E(Y|Z = z_0) = E(Y(t_1) - Y(t_0)|\boldsymbol{S} \in \{\boldsymbol{s}_4, \boldsymbol{s}_5, \boldsymbol{s}_6\})P(\boldsymbol{S} \in \{\boldsymbol{s}_4, \boldsymbol{s}_5, \boldsymbol{s}_6\}) \tag{111}$$
$$+ E(Y(t_2) - Y(t_0)|\boldsymbol{S} \in \{\boldsymbol{s}_7, \boldsymbol{s}_8, \boldsymbol{s}_9\})P(\boldsymbol{S} \in \{\boldsymbol{s}_7, \boldsymbol{s}_8, \boldsymbol{s}_8\}) \tag{112}$$
$$+ E(Y(t_0) - Y(t_2)|\boldsymbol{S} = \boldsymbol{s}_13)P(\boldsymbol{S} = \boldsymbol{s}_{13}) \tag{113}$$
$$+ E(Y(t_2) - Y(t_1)|\boldsymbol{S} = \boldsymbol{s}_14)P(\boldsymbol{S} = \boldsymbol{s}_{14}). \tag{114}$$

Voucher effect (111) lacks causal interpretation in terms of treatment effects as it contains conflicting terms. It includes the treatment effect $Y(t_2) - Y(t_0)$ for response-types $\boldsymbol{s}_7, \boldsymbol{s}_8, \boldsymbol{s}_9$ but also the opposite effect $Y(t_0) - Y(t_2)$ for response-type $\boldsymbol{s}_{13}$. The same problem occurs to the remaining voucher effects, $E(Y|Z = z_2) - E(Y|Z = z_1)$ and $E(Y|Z = z_2) - E(Y|Z = z_1)$.

KLM investigates the estimation that use the IV indicators $D_{z_1} = \mathbf{1}[Z = z_1]$, $D_{z_2} = \mathbf{1}[Z = z_2]$ as instruments for the choice indicators $D_{t_1} = \mathbf{1}[T = t_1]$, $D_{t_2} = \mathbf{1}[T = t_2]$ in the following regression:

$$Y = \alpha + D_{t_1}\beta_1 + D_{t_2}\beta_2 + \epsilon. \tag{115}$$

They show that the estimates for β_1 and β_2 lack causal interpretation in terms of treatment effects. This section corroborates their finding by showing that the estimates of β_1 and β_2 can be expressed as a weighted average of the three voucher effects discussed above, which do not have a causal interpretation in terms of treatment effects.

It is easy to show that estimates $\hat{\beta}_1, \hat{\beta}_2$ are the sample analog of the following parameters:

Table A.5: Elimination of Response-types of the Three-choice Model with a Parallel Design

Panel A	**All 27 Possible Response-types**																										
Counterfactual Choices	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
$T_i(z_0)$	t_0	t_0	t_0	t_0	t_0	t_0	t_0	t_0	t_0	t_1	t_1	t_1	t_1	t_1	t_1	t_1	t_1	t_1	t_2	t_2	t_2	t_2	t_2	t_2	t_2	t_2	t_2
$T_i(z_1)$	t_0	t_0	t_0	t_1	t_1	t_1	t_2	t_2	t_2	t_0	t_0	t_0	t_1	t_1	t_1	t_2	t_2	t_2	t_0	t_0	t_0	t_1	t_1	t_1	t_2	t_2	t_2
$T_i(z_2)$	t_0	t_1	t_2	t_0	t_1	t_2	t_0	t_1	t_2	t_0	t_1	t_2	t_0	t_1	t_2	t_0	t_1	t_2	t_0	t_1	t_2	t_0	t_1	t_2	t_0	t_1	t_2
Panel B	**Response-type Eliminated by Monotonicity Conditions (108)–(109)**																										
Condition 1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓	✓	✗	✗	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓
Condition 2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✓	✗	✗	✓	✗	✗	✓
Not Eliminated	1	2	3	4	5	6	7	8	9				13	14	15						21			24			27
Panel C	**Response-type Eliminated by Revealed Preference Analysis**																										
Restriction 1	✓	✗	✓	✓	✗	✓	✗	✗	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Restriction 2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗	✓	✓	✗	✗	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓
Restriction 3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗	✗	✓	✗	✗	✓
Restriction 4	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓	✓	✓	✓	✓	✗	✗	✓
Restriction 5	✓	✗	✓	✓	✗	✓	✓	✗	✓	✓	✗	✓	✓	✓	✓	✓	✗	✓	✓	✗	✓	✓	✗	✓	✓	✗	✓
Not Eliminated	1		3	4		6								14	15									24			27

Panel A lists the 27 possible response-types that the response variable $S_i = [T_i(z_0)\,,\, T_i(z_1)\,,\, T_i(z_2)]$ can take. Rows present the counterfactual choices an agent i could choose if it were assigned to z_0, z_1, and z_2 respectively. Columns present all the values of response-type as choices range over $supp(T) = \{t_0, t_1, t_2\}$. Panel B describes an elimination process based on the two monotonicity conditions (108)–(109). These criteria are also stated in Panel B′ below. Panel C describes an elimination process based on the seven choice restrictions generated by the revealed preference analysis. These choice restrictions are also displayed in Panel C′ below.

Panel B′ – Monotonicity Relations

Monotonicity Condition 1	$\mathbf{1}[T_i(z_0) = t_1]$	$\leq$	$\mathbf{1}[T_i(z_1) = t_1]$
Monotonicity Condition 2	$\mathbf{1}[T_i(z_0) = t_2]$	$\leq$	$\mathbf{1}[T_i(z_2) = t_2]$

Panel C′ – Choice Restrictions

Choice Restriction 1	$T_i(z_0) = t_0$	$\Rightarrow$	$T_i(z_1) \neq t_2$	and	$T_i(z_2) \neq t_1$
Choice Restriction 2	$T_i(z_0) = t_1$	$\Rightarrow$	$T_i(z_1) = t_1$	and	$T_i(z_2) \neq t_0$
Choice Restriction 3	$T_i(z_0) = t_2$	$\Rightarrow$	$T_i(z_1) \neq t_0$	and	$T_i(z_2) = t_2$
Choice Restriction 4	$T_i(z_1) = t_2$	$\Rightarrow$	$T_i(z_0) = t_2$	and	$T_i(z_2) = t_2$
Choice Restriction 5	$T_i(z_2) = t_1$	$\Rightarrow$	$T_i(z_0) = t_1$	and	$T_i(z_1) = t_1$

Check mark ✓ indicates that the response-type displayed by the top column of the table does not violate the choice restriction denoted by the panel row. Cross sign ✗indicates that the response-type violates the choice restriction and should be eliminated. The last row in each panel presents the response-types that survive the elimination process.

$$\beta_1 = \frac{cov(D_{t_2}, D_{z_2})cov(D_{z_2}, Y) - cov(D_{t_2}, D_{z_1})cov(D_{z_2}, Y)}{cov(D_{t_1}, D_{z_1})cov(D_{t_2}, D_{z_2}) - cov(D_{t_1}, D_{z_2})cov(D_{t_2}, D_{z_1})}, \tag{116}$$

$$\beta_2 = \frac{cov(D_{t_1}, D_{z_1})cov(D_{z_2}, Y) - cov(D_{t_1}, D_{z_2})cov(D_{z_1}, Y)}{cov(D_{t_1}, D_{z_1})cov(D_{t_2}, D_{z_2}) - cov(D_{t_1}, D_{z_2})cov(D_{t_2}, D_{z_1})}. \tag{117}$$

Revealed Preference Analysis

Revealed preference analysis is more effective in eliminating response-types than monotonicity conditions (108)–(109). Table A.6 applies the WARP restriction in **P.2** to the incentive matrix (107). There are 22 binding restrictions that can be summarized into the five choice restrictions in Table A.7. That is to say that the restrictions in Table A.7 summarise a subset of the restrictions in Table A.6. The remaining restrictions do not eliminate any additional response-types that are not already eliminated by the five restrictions in Table A.7.

The WARP restriction in **P.2** translates the incentive matrix 29 into the five choice restrictions:

1	$T_i(z_0) = t_0$	$\Rightarrow$	$T_i(z_1) \neq t_2$	and	$T_i(z_2) \neq t_1$
2	$T_i(z_0) = t_1$	$\Rightarrow$	$T_i(z_1) = t_1$	and	$T_i(z_2) \neq t_0$
3	$T_i(z_0) = t_2$	$\Rightarrow$	$T_i(z_1) \neq t_0$	and	$T_i(z_2) = t_2$
4	$T_i(z_1) = t_2$	$\Rightarrow$	$T_i(z_0) = t_2$	and	$T_i(z_2) = t_2$
5	$T_i(z_2) = t_1$	$\Rightarrow$	$T_i(z_0) = t_1$	and	$T_i(z_1) = t_1$

The first restriction consider an agent that chooses t_0 under z_0. This mean that t_0 is revealed preferred to t_1 and t_2 when no incentives are available. The IV value z_1 offers no incentives towards t_2, hence t_2 is not chosen under z_1. In same token, z_2 does not incentivize t_1 and therefore t_1 is not chosen. Panel C of Table (A.5) shows that these five restrictions eliminate 19 out of the 27 possible response-types. The columns of response matrix (118) display the eight remaining response-types:

$$\boldsymbol{R} = \begin{array}{c} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 \end{matrix} \\ \begin{bmatrix} t_1 & t_1 & t_0 & t_0 & t_2 & t_0 & t_0 & t_2 \\ t_1 & t_1 & t_1 & t_1 & t_1 & t_0 & t_0 & t_2 \\ t_1 & t_2 & t_0 & t_2 & t_2 & t_0 & t_2 & t_2 \end{bmatrix} \end{array} \begin{matrix} T_i(z_0) \\ T_i(z_1) \\ T_i(z_2) \end{matrix} \tag{118}$$

The main paper shows that response matrix above satisfies the monotonicity condition of Angrist and Imbens (1995).

Table A.6: Choice Restrictions Due to WARP

#	Revealed Choice	Incentive Inequalities				Choice Statement
	$T_i(z)=t$	$\boldsymbol{L}[z',t']-\boldsymbol{L}[z,t'] \leq 0 \leq \boldsymbol{L}[z',t]-\boldsymbol{L}[z,t]$				$T(z') \neq t'$
1	$T_i(z_0)=t_0,$	$\boldsymbol{L}[z_2,t_1]-\boldsymbol{L}[z_0,t_1]=0$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_2,t_0]-\boldsymbol{L}[z_0,t_0]$	$T_i(z_2) \neq t_1$
2	$T_i(z_0)=t_0,$	$\boldsymbol{L}[z_1,t_2]-\boldsymbol{L}[z_0,t_2]=0$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_1,t_0]-\boldsymbol{L}[z_0,t_0]$	$T_i(z_1) \neq t_2$
3	$T_i(z_0)=t_1,$	$\boldsymbol{L}[z_1,t_0]-\boldsymbol{L}[z_0,t_0]=0$	$\leq$	0	$\leq$ $1=\boldsymbol{L}[z_1,t_1]-\boldsymbol{L}[z_0,t_1]$	$T_i(z_1) \neq t_0$
4	$T_i(z_0)=t_1,$	$\boldsymbol{L}[z_2,t_0]-\boldsymbol{L}[z_0,t_0]=0$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_2,t_1]-\boldsymbol{L}[z_0,t_1]$	$T_i(z_2) \neq t_0$
5	$T_i(z_0)=t_1,$	$\boldsymbol{L}[z_1,t_2]-\boldsymbol{L}[z_0,t_2]=0$	$\leq$	0	$\leq$ $1=\boldsymbol{L}[z_1,t_1]-\boldsymbol{L}[z_0,t_1]$	$T_i(z_1) \neq t_2$
6	$T_i(z_0)=t_2,$	$\boldsymbol{L}[z_1,t_0]-\boldsymbol{L}[z_0,t_0]=0$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_1,t_2]-\boldsymbol{L}[z_0,t_2]$	$T_i(z_1) \neq t_0$
7	$T_i(z_0)=t_2,$	$\boldsymbol{L}[z_2,t_0]-\boldsymbol{L}[z_0,t_0]=0$	$\leq$	0	$\leq$ $1=\boldsymbol{L}[z_2,t_2]-\boldsymbol{L}[z_0,t_2]$	$T_i(z_2) \neq t_0$
8	$T_i(z_0)=t_2,$	$\boldsymbol{L}[z_2,t_1]-\boldsymbol{L}[z_0,t_1]=0$	$\leq$	0	$\leq$ $1=\boldsymbol{L}[z_2,t_2]-\boldsymbol{L}[z_0,t_2]$	$T_i(z_2) \neq t_1$
9	$T_i(z_1)=t_0,$	$\boldsymbol{L}[z_0,t_1]-\boldsymbol{L}[z_1,t_1]=-1$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_0,t_0]-\boldsymbol{L}[z_1,t_0]$	$T_i(z_0) \neq t_1$
10	$T_i(z_1)=t_0,$	$\boldsymbol{L}[z_2,t_1]-\boldsymbol{L}[z_1,t_1]=-1$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_2,t_0]-\boldsymbol{L}[z_1,t_0]$	$T_i(z_2) \neq t_1$
11	$T_i(z_1)=t_0,$	$\boldsymbol{L}[z_0,t_2]-\boldsymbol{L}[z_1,t_2]=0$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_0,t_0]-\boldsymbol{L}[z_1,t_0]$	$T_i(z_0) \neq t_2$
12	$T_i(z_1)=t_2,$	$\boldsymbol{L}[z_0,t_0]-\boldsymbol{L}[z_1,t_0]=0$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_0,t_2]-\boldsymbol{L}[z_1,t_2]$	$T_i(z_0) \neq t_0$
13	$T_i(z_1)=t_2,$	$\boldsymbol{L}[z_2,t_0]-\boldsymbol{L}[z_1,t_0]=0$	$\leq$	0	$\leq$ $1=\boldsymbol{L}[z_2,t_2]-\boldsymbol{L}[z_1,t_2]$	$T_i(z_2) \neq t_0$
14	$T_i(z_1)=t_2,$	$\boldsymbol{L}[z_0,t_1]-\boldsymbol{L}[z_1,t_1]=-1$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_0,t_2]-\boldsymbol{L}[z_1,t_2]$	$T_i(z_0) \neq t_1$
15	$T_i(z_1)=t_2,$	$\boldsymbol{L}[z_2,t_1]-\boldsymbol{L}[z_1,t_1]=-1$	$\leq$	0	$\leq$ $1=\boldsymbol{L}[z_2,t_2]-\boldsymbol{L}[z_1,t_2]$	$T_i(z_2) \neq t_1$
16	$T_i(z_2)=t_0,$	$\boldsymbol{L}[z_0,t_1]-\boldsymbol{L}[z_2,t_1]=0$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_0,t_0]-\boldsymbol{L}[z_2,t_0]$	$T_i(z_0) \neq t_1$
17	$T_i(z_2)=t_0,$	$\boldsymbol{L}[z_0,t_2]-\boldsymbol{L}[z_2,t_2]=-1$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_0,t_0]-\boldsymbol{L}[z_2,t_0]$	$T_i(z_0) \neq t_2$
18	$T_i(z_2)=t_0,$	$\boldsymbol{L}[z_1,t_2]-\boldsymbol{L}[z_2,t_2]=-1$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_1,t_0]-\boldsymbol{L}[z_2,t_0]$	$T_i(z_1) \neq t_2$
19	$T_i(z_2)=t_1,$	$\boldsymbol{L}[z_0,t_0]-\boldsymbol{L}[z_2,t_0]=0$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_0,t_1]-\boldsymbol{L}[z_2,t_1]$	$T_i(z_0) \neq t_0$
20	$T_i(z_2)=t_1,$	$\boldsymbol{L}[z_1,t_0]-\boldsymbol{L}[z_2,t_0]=0$	$\leq$	0	$\leq$ $1=\boldsymbol{L}[z_1,t_1]-\boldsymbol{L}[z_2,t_1]$	$T_i(z_1) \neq t_0$
21	$T_i(z_2)=t_1,$	$\boldsymbol{L}[z_0,t_2]-\boldsymbol{L}[z_2,t_2]=-1$	$\leq$	0	$\leq$ $0=\boldsymbol{L}[z_0,t_1]-\boldsymbol{L}[z_2,t_1]$	$T_i(z_0) \neq t_2$
22	$T_i(z_2)=t_1,$	$\boldsymbol{L}[z_1,t_2]-\boldsymbol{L}[z_2,t_2]=-1$	$\leq$	0	$\leq$ $1=\boldsymbol{L}[z_1,t_1]-\boldsymbol{L}[z_2,t_1]$	$T_i(z_1) \neq t_2$

This table displays the binding choice restrictions generated by the WARP restriction below

$$\text{If} \quad T_i(z)=t \quad \text{and} \quad \boldsymbol{L}[z',t']-\boldsymbol{L}[z,t'] \leq 0 \leq \boldsymbol{L}[z',t]-\boldsymbol{L}[z,t] \quad \text{then} \quad T_i(z') \neq t'.$$

when applied to the MTO incentive matrix:

$$\text{MTO Incentive Matrix } \boldsymbol{L} = \begin{array}{c} \\ \end{array}\begin{matrix} t_0 & t_1 & t_2 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} z_0 \\ z_1 \\ z_2 \end{matrix}$$

Table A.7: Summary of Choice Restrictions generated by applying WARP to the Parallel Design Model in Table A.6

#	Choice Restrictions				
1,2	$T_i(z_0)=t_0$	$\Rightarrow$	$T_i(z_1) \neq t_2$	and	$T_i(z_2) \neq t_1$
3,4,5	$T_i(z_0)=t_1$	$\Rightarrow$	$T_i(z_1) = t_1$	and	$T_i(z_2) \neq t_0$
6,7,8	$T_i(z_0)=t_2$	$\Rightarrow$	$T_i(z_1) \neq t_0$	and	$T_i(z_2) = t_2$
12,13,14,15	$T_i(z_1)=t_2$	$\Rightarrow$	$T_i(z_0) = t_2$	and	$T_i(z_2) = t_2$
19,20,21,22	$T_i(z_2)=t_1$	$\Rightarrow$	$T_i(z_0) = t_1$	and	$T_i(z_1) = t_1$

F MTO Incentives do not Justify Ordered Choices

A natural inquiry is whether it is possible to model neighborhood choices in MTO as an ordered choice model. Viewing the treatment as ordered is appealing because it relates to the well-known monotonicity condition of Angrist and Imbens (1995) described below:

$$\text{For any } z, z' \ , T_i(z) \leq T_i(z') \forall \, i \text{ or } T_i(z) \geq T_i(z') \forall \, i. \tag{119}$$

in (119), (Vytlacil, 2006) demonstrates that the monotonicity condition 119 is equivalent to assuming an ordered choice model. For sake of clarity, condition (119) is termed ordered monotonicity henceforward.

Ordered monotonicity (119) benefits from well-establish literature in policy evaluation. In particular, Angrist and Imbens (1995) has shown that the standard 2SLS regression evaluates an interpretable causal parameter under (119). Unfortunately, ordered monotonicity (119) is not compatible with MTO incentives.

Ordered monotonicity (119) is equivalent to state that there exist a sequence of instrumental variables $z_1, ..., z_J$ and such that $T_i(z_1) < ... < T_i(z_J)$ for all agents $i \in \mathcal{I}$. If (119) were true for MTO, we would be able to relabel the instrumental values and neighborhood choices of MTO, say $supp(Z) = \{z_1, z_2, z_3\}$, and $T \in \{1, 2, 3\}$, such that

$$T_i(z_1) \leq T_i(z_2) \leq T_i(z_3) \text{ holds for all } i \in \mathcal{I}. \tag{120}$$

Unfortunately, condition (120) does not hold regardless of how we label instrumental values and neighborhood choices. To see this, let the instrumental values z_c, z_8, z_e be relabeled as z_1, z_2, z_3 and the neighborhood choices z_h, z_m, z_l as $1, 2, 3$. In this notation, the MTO response matrix is given by:

$$\text{Relabeled MTO Response Matrix}: \quad \boldsymbol{R} = \begin{array}{c} \begin{matrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 3 & 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 & 3 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 & 3 & 1 \end{bmatrix} \end{array} \begin{matrix} T_i(z_1) \\ T_i(z_2) \\ T_i(z_3) \end{matrix} \tag{121}$$

The response-types $\boldsymbol{s}_1$ until $\boldsymbol{s}_6$ are weakly increasing, which comply with the monotonicity condition $T_i(z_1) \leq T_i(z_2) \leq T_i(z_3)$. Response-type $\boldsymbol{s}_7$ however violates this condition as $T_i(z_2) > T_i(z_3)$. Switching the second and third rows of (121) would make $\boldsymbol{s}_7$ comply with the monotonicity criteria (120), but $\boldsymbol{s}_4$ would violate it. It is easy to see that the monotonicity condition would not be satisfied by relabeling the neighborhood choices either.

F.1 Incentives that Justify Ordered Monotonicity (119)

In the paper called "economics of monotonicity," I investigate which incentive schemes justify the ordered choice models. The paper describe the necessary and sufficient condition that the incentive matrix must have to generate the monotonicity condition of Angrist and Imbens (1995) in (119) under WARP and normal choices.

The are several incentive schemes that justify the ordered monotonicity (120). Let the incentive matrix $\boldsymbol{L}$ be the $J \times K$ matrix that characterises the incentives induced by instrumental values in $Z \in \{z_1, ..., z_J\}$, toward choices in $T \in \{1, ..., K\}$. The matrix input $\boldsymbol{L}[z_j, k]$ denotes the incentive for choosing choice $k \in \{1, ..., K\}$ when assigned to instrumental variable $z_j\{z_1, ..., z_J\}$. One incentive scheme that generates the ordered choice models is the presence of increasing incentive increments, that is:

$$\boldsymbol{L}[z_{j+1}, t_k] - \boldsymbol{L}[z_j, t_k] < \boldsymbol{L}[z_{j+1}, t_{k+1}] - \boldsymbol{L}[z_j, t_{k+1}] \text{ for } j \in \{1, ..., J-1\} \text{ and } k \in \{1, ..., K-1\}. \tag{122}$$

Examples of such incentives for three-choice model and a three-valued instrument of MTO are:

$$\boldsymbol{L}_1 = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix} \end{array} \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array}, \quad \boldsymbol{L}_2 = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix} \end{array} \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array}, or \quad \boldsymbol{L}_3 = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 4 & 16 & 64 \end{bmatrix} \end{array} \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array} \tag{123}$$

The combination of WARP and Normal Choice criteria of Section 3 generates the following choice restriction:

$$\text{If } T_i(z) = t \text{ and } \boldsymbol{L}[z', t'] - \boldsymbol{L}[z, t'] \leq \boldsymbol{L}[z', t] - \boldsymbol{L}[z, t] \text{ then } T_i(z') \neq t'. \tag{124}$$

Choice Rule 124 is intuitive. It states that if an agent i chooses t instead of t' under z, and z' offers greater incentives towards t than t', then agent i will not choose t' under z'. Applying Choice Rule (124) to any of the incentive matrices $\boldsymbol{L}_1$, $\boldsymbol{L}_2$ or $\boldsymbol{L}_3$ in (123) generates the following response matrix:

$$\text{MTO Response Matrix:} \quad \boldsymbol{R} = \begin{array}{c} \begin{array}{cccccccccc} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \boldsymbol{s}_3 & \boldsymbol{s}_4 & \boldsymbol{s}_5 & \boldsymbol{s}_6 & \boldsymbol{s}_7 & \boldsymbol{s}_8 & \boldsymbol{s}_9 & \boldsymbol{s}_{10} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 \\ 1 & 1 & 1 & 2 & 2 & 3 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 2 & 3 & 3 & 2 & 3 & 3 & 3 \end{bmatrix} \end{array} \begin{array}{c} T_i(z_1) \\ T_i(z_2) \\ T_i(z_3) \end{array} \tag{125}$$

Response matrix (125) contains all the admissible response-types that satisfy the monotonicity condition $T_i(z_1) \leq T_i(z_2) \leq T_i(z_3)$. Indeed, the choices in each of the response-types of (125) are weakly increasing. Moreover, there is no response-type other than those in (125) that satisfy the ordered monotonicity 120.

F.2 What is Identified by Ordered Monotonicity (119)?

As mentioned, Response matrix 125 is obtained by assuming the monotonicity assumption 120 of Angrist and Imbens (1995) in the case of a three choice model with a three-valued instrument. The response matrix has 10 response-types comprising of 18 counterfactual outcomes condition on response-types. For instance, there are six response types that take treatment value the treatment $T = 1$, namely, $\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{s}_3, \boldsymbol{s}_4, \boldsymbol{s}_5$ and $\boldsymbol{s}_6$. There are also six response types that take treatment value the treatment $T = 2 : \boldsymbol{s}_2, \boldsymbol{s}_4, \boldsymbol{s}_5, \boldsymbol{s}_7, \boldsymbol{s}_8$ and $\boldsymbol{s}_9$. Finally, there are six response types that take treatment value the treatment $T = 3 : \boldsymbol{s}_3, \boldsymbol{s}_5, \boldsymbol{s}_6, \boldsymbol{s}_8, \boldsymbol{s}_9$ and $\boldsymbol{s}_{10}$.

The identification of response-type probabilities and counterfactual outcomes is based on

Equation 34 provides the necessary and sufficient criteria to examine the identification of counterfactual outcomes. The criteria can also be use to examine the identification of response-type probabilities (by setting the outcome to one). The application of the criteria to the response matrix 125 enables the identification of the following causal parameters:

1. The following response-type probabilities are identified:

$$P(S=s_1), P(S=s_{10}), P(S\in\{s_2,s_3\}), P(S\in\{s_6,s_9\}), \text{ and } P(S\in\{s_4,s_5,s_7,s_8\}).$$

2. The following counterfactual outcome expectations are identified:

$$\begin{aligned}&E(Y(1)|\boldsymbol{S}=\boldsymbol{s}_1), E(Y(1)|\boldsymbol{S}\in\{\boldsymbol{s}_2,\boldsymbol{s}_3\}), \text{ and } E(Y(1)|\boldsymbol{S}\in\{\boldsymbol{s}_4,\boldsymbol{s}_5,\boldsymbol{s}_6\})\\&E(Y(2)|\boldsymbol{S}\in\{\boldsymbol{s}_2,\boldsymbol{s}_4,\boldsymbol{s}_7\}),\ E(Y(2)|\boldsymbol{S}\in\{\boldsymbol{s}_7,\boldsymbol{s}_8,\boldsymbol{s}_9\}), \text{ and } E(Y(2)|\boldsymbol{S}\in\{\boldsymbol{s}_4,\boldsymbol{s}_5,\boldsymbol{s}_7,\boldsymbol{s}_8\})\\&E(Y(3)|\boldsymbol{S}=\boldsymbol{s}_{10}), E(Y(3)|\boldsymbol{S}\in\{\boldsymbol{s}_6,\boldsymbol{s}_9\}), \text{ and } E(Y(3)|\boldsymbol{S}\in\{\boldsymbol{s}_3,\boldsymbol{s}_5,\boldsymbol{s}_8\}).\end{aligned}$$

Note that there are only two response-type probabilities that are point-identified: $P(S=s_1)$, and $P(S=s_{10})$. Moreover, only two out of the 18 counterfactual outcomes listed above are point-identified: $E(Y(1)|\boldsymbol{S}=\boldsymbol{s}_1)$, and $E(Y(3)|\boldsymbol{S}=\boldsymbol{s}_{10})$. The number of identified parameters for MTO are sharply different. The response matrix (121) has seven response-types, all response-type probabilities are point-identified, and six out of 12 counterfactual outcomes are point-identified.

G Exploring the Benefits of the Response Matrix

The response matrix summarizes the necessary and sufficient information to investigate the non-parametric identification of response-type probabilities and counterfactual outcomes. This section discusses three topics regarding the usage of the response matrix. Section G.1 clarifies how the response matrix enables us to map counterfactuals and observed data. Section G.2 discusses the identification of counterfactual outcomes. The section provides closed-form solutions for each identified counterfactual. Section G.3 explains how to estimate the identified counterfactual outcomes using stands 2SLS regressions.

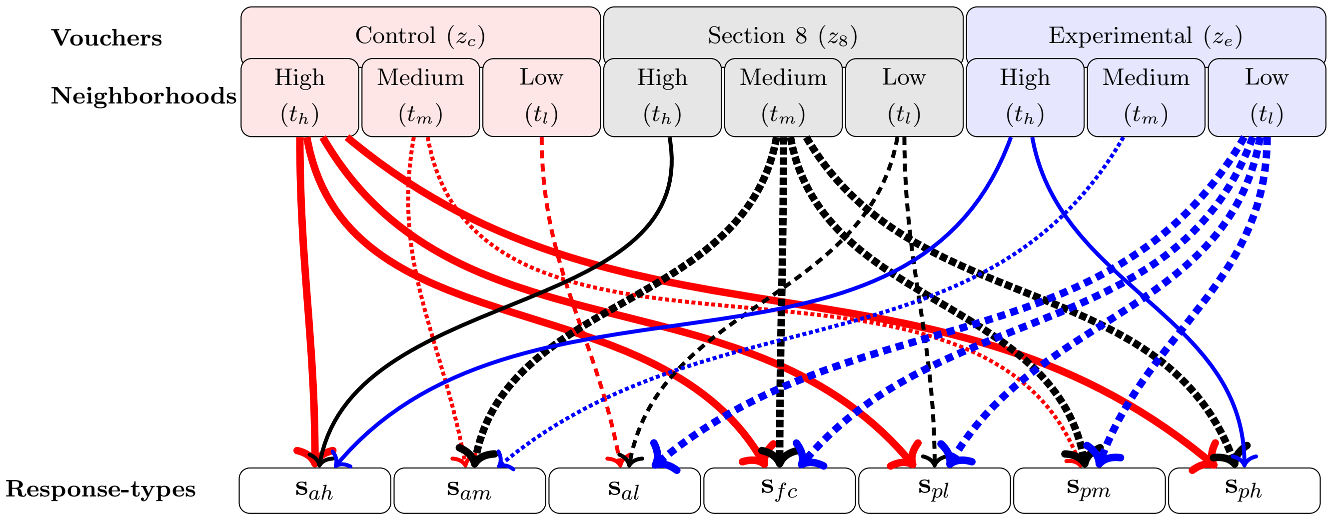
G.1 Mapping Counterfactual outcomes with Observed Data

The matrix determines a mapping between observed choices and latent response-types. For instance, the first row of the response matrix (28) lists the choices for control z_c. Families that choose t_l under z_c can only be low-poverty always-takers $\boldsymbol{s}_{al}$. Families that choose t_m under z_c are a mixture of $\boldsymbol{s}_{al}$ and $\boldsymbol{s}_{al}$, while those who choose t_l under z_c can be of four types: $\boldsymbol{s}_{ah}, \boldsymbol{s}_{am}, \boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}$ or $, \boldsymbol{s}_{ph}$. Figure A.7 displays the mapping generated by the response matrix (28). The identification of causal parameters consists of disentangling this mapping.

G.2 Interpreting Identification Results

Equation (8) is central to the identification analysis. It shows that the indicator $\mathbf{1}[T=t|\boldsymbol{S}=\boldsymbol{s}, Z=z]$ connects observed data, i.e., the expectation of the outcome multiplied by the choice indicator, with the unobserved parameters we seek to identify, i.e. potential outcomes $E(Y(t)|\boldsymbol{S}=\boldsymbol{s})$ and response-type probabilities $P(\boldsymbol{S}=\boldsymbol{s})$. The identification of counterfactual parameters consists of expressing the unobserved variables in the right-hand side of (8) in terms of the observed variables

Figure A.7: **From Observed Vouchers and Choices to Unobserved Response-types**



	$\mathbf{s}_{ah}$	$\mathbf{s}_{am}$	$\mathbf{s}_{al}$	$\mathbf{s}_{fc}$	$\mathbf{s}_{pl}$	$\mathbf{s}_{pm}$	$\mathbf{s}_{ph}$
$T(z_c)$	t_h	t_m	t_l	t_h	t_h	t_m	t_h
$T(z_8)$	t_h	t_m	t_l	t_m	t_l	t_m	t_m
$T(z_e)$	t_h	t_m	t_l	t_l	t_l	t_l	t_h

This figure describes how voucher assignments and neighborhood choices map into the MTO response-types. There are three possible voucher assignments: Control (z_c), Section 8 (z_8), or Experimental (z_e). There are three neighborhood choices: high-poverty neighborhood (t_h), medium-poverty neighborhood (t_m) or low-poverty neighborhood (t_l). The combination of voucher assignment and neighborhood choice generate nine possibilities. There are seven response-types according to the response matrix $\boldsymbol{R}$ in (28). These response-types are denoted by $\boldsymbol{s}_{ah}, \boldsymbol{s}_{am}, \boldsymbol{s}_{al}, \boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}, \boldsymbol{s}_{pm}, \boldsymbol{s}_{pl}$. The mapping between the voucher assignments and neighborhood choices into response-types is represented by connecting lines. Solid lines denote the choice of high-poverty neighborhood. Dotted lines denote the choice of medium-poverty neighborhood. Dashed lines denote the choice of low-poverty neighborhood. Bold lines refer to the most frequent neighborhood choice for each voucher assignment.

of the left-hand side. This problem is best examined by expressing equation (8) in matrix form:

$$\boldsymbol{Q}_Z(t) \odot \boldsymbol{P}_Z(t) = \boldsymbol{B}_t \cdot \Big(\boldsymbol{Q}_S(t) \odot \boldsymbol{P}_S\Big); \quad t \in \{t_h, t_m, t_l\}, \tag{126}$$

where
$$\begin{aligned}
\boldsymbol{P}_Z(t) &= [P(T=t|Z=z_c), P(T=t|Z=z_8), P(T=t|Z=z_e)]', \\
\boldsymbol{Q}_Z(t) &= [E(Y|T=t, Z=z_c), E(Y|T=t, Z=z_8), E(Y|T=t, Z=z_e)]', \\
\boldsymbol{P}_S &= [P(\boldsymbol{S}=\boldsymbol{s}_{ah}), P(\boldsymbol{S}=\boldsymbol{s}_{am}), P(\boldsymbol{S}=\boldsymbol{s}_{al}), P(\boldsymbol{S}=\boldsymbol{s}_{fc}), P(\boldsymbol{S}=\boldsymbol{s}_{pl}), P(\boldsymbol{S}=\boldsymbol{s}_{pm}), P(\boldsymbol{S}=\boldsymbol{s}_{ph})]', \\
\boldsymbol{Q}_S(t) &= [E(Y(t)|\boldsymbol{S}=\boldsymbol{s}_{ah}), ..., E(Y(t)|\boldsymbol{S}=\boldsymbol{s}_{ph})]', \\
\boldsymbol{B}_t &= \mathbf{1}[\boldsymbol{R}=t]; t \in \{t_l, t_m, t_h\}.
\end{aligned}$$

$\boldsymbol{P}_Z(t)$ denotes the observed vector of propensity scores. $\boldsymbol{Q}_Z(t)$ denotes the observed vector of conditional outcomes. $\boldsymbol{P}_S$ is the 7×1 vector of response-type probabilities. $\boldsymbol{Q}_S(t)$ is the unobserved vector of counterfactual outcome means. $\boldsymbol{B}_t = \mathbf{1}[\boldsymbol{R}=t]$ denotes the 3×7 binary matrix that indicates which elements in $\boldsymbol{R}$ are equal to $t \in \{t_h, t_m, t_l\}$ and $\odot$ denotes the Hadamard product (element-wise multiplication).

The the binary matrices $\boldsymbol{B}_t = \mathbf{1}[\boldsymbol{R}=t]$ for t_l, t_m, and t_h are displayed in equations (128),(129) and (130) respectively. It is useful to decompose each binary matrix $\boldsymbol{B}_t$ into $\boldsymbol{B}_t = \boldsymbol{C}_t \cdot \boldsymbol{A}_t$, where $\boldsymbol{C}_t$ is the array the non-zero columns of $\boldsymbol{B}_t$ and $\boldsymbol{A}_t$ is a mapping between the vectors in $\boldsymbol{C}_t$ and $\boldsymbol{B}_t$.

Specifically, the response matrix $\boldsymbol{R}$ is decomposed as:

$$\boldsymbol{R} \equiv \sum_{t \in supp(T)} t \cdot \boldsymbol{B}_t = \sum_{t \in supp(T)} t \cdot \boldsymbol{C}_t \boldsymbol{A}_t, \tag{127}$$

where $\boldsymbol{B}_t, \boldsymbol{C}_t, \boldsymbol{A}_t$ for $t \in \{t_h, t_m, t_h\}$ are given by:

$$\boldsymbol{B}_{t_h} = \begin{array}{c} \begin{matrix} \boldsymbol{s}_{ah} & \boldsymbol{s}_{am} & \boldsymbol{s}_{al} & \boldsymbol{s}_{fc} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{ph} \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = \underbrace{\begin{array}{c} \begin{matrix} \boldsymbol{s}_{fc}, \boldsymbol{s}_{pl} & \boldsymbol{s}_{ph} & \boldsymbol{s}_{ah} \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}}_{\boldsymbol{C}_{t_h}} \cdot \underbrace{\begin{array}{c} \begin{matrix} \boldsymbol{s}_{ah} & \boldsymbol{s}_{am} & \boldsymbol{s}_{al} & \boldsymbol{s}_{fc} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{ph} \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}}_{\boldsymbol{A}_{t_h}} \tag{128}$$

$$\boldsymbol{B}_{t_m} = \begin{array}{c} \begin{matrix} \boldsymbol{s}_{ah} & \boldsymbol{s}_{am} & \boldsymbol{s}_{al} & \boldsymbol{s}_{fc} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{ph} \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} = \underbrace{\begin{array}{c} \begin{matrix} \boldsymbol{s}_{fc}, \boldsymbol{s}_{ph} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{am} \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}}_{\boldsymbol{C}_{t_m}} \cdot \underbrace{\begin{array}{c} \begin{matrix} \boldsymbol{s}_{ah} & \boldsymbol{s}_{am} & \boldsymbol{s}_{al} & \boldsymbol{s}_{fc} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{ph} \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}}_{\boldsymbol{A}_{t_m}} \tag{129}$$

$$\boldsymbol{B}_{t_l} = \begin{array}{c} \begin{matrix} \boldsymbol{s}_{ah} & \boldsymbol{s}_{am} & \boldsymbol{s}_{al} & \boldsymbol{s}_{fc} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{ph} \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array} = \underbrace{\begin{array}{c} \begin{matrix} \boldsymbol{s}_{fc}, \boldsymbol{s}_{pm} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{al} \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}}_{\boldsymbol{C}_{t_l}} \cdot \underbrace{\begin{array}{c} \begin{matrix} \boldsymbol{s}_{ah} & \boldsymbol{s}_{am} & \boldsymbol{s}_{al} & \boldsymbol{s}_{fc} & \boldsymbol{s}_{pl} & \boldsymbol{s}_{pm} & \boldsymbol{s}_{ph} \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}}_{\boldsymbol{A}_{t_l}} \tag{130}$$

Matrices $\boldsymbol{C}_t$ and $\boldsymbol{A}_t$ are used to generate a closed-form solution for the nonparametric identification of counterfactual outcomes. The main paper shows that for each neighborhood choice, we can reorder the columns and rows of the response matrix $\boldsymbol{R}$ to generate a lower-triangular matrix (see equations (35) and (42)). This triangular property means that for each t we have that:

$$\text{for any} z, z' \in supp(Z), \text{ we have that } \boldsymbol{B}_t[z, \boldsymbol{s}] \leq \boldsymbol{B}_t[z', \boldsymbol{s}] \forall\, \boldsymbol{s} \text{ or } \boldsymbol{B}_t[z, \boldsymbol{s}] \geq \boldsymbol{B}_t[z', \boldsymbol{s}] \forall\, \boldsymbol{s}. \tag{131}$$

Equation (131) is equivalent to state that there exists a sequence of of IV-values $z_1^{(t)}, ...z_N^{(t)}$ of the values in $supp(Z) \equiv \{z_1, ..., z_N\}$ such that:

$$\boldsymbol{B}_t[z_k^{(t)}, \boldsymbol{s}] \leq \boldsymbol{B}_t[z_{k+1}^{(t)}, \boldsymbol{s}] \;\forall\, \boldsymbol{s} \in supp(\boldsymbol{S}); k = 1, .., N-1. \tag{132}$$

This triangular property implies that matrices $\boldsymbol{C}_t$ in the decompositions (128)–(130) are of full row-rank. We can then use the generalized solution of linear equations in Magnus and Neudecker (1999) to identify counterfactual outcomes the following equation:

$$\Big(\boldsymbol{A}_t(\boldsymbol{Q}_S(t) \odot \boldsymbol{P}_S\Big) \div \Big(\boldsymbol{A}_t \boldsymbol{P}_S\Big) = \Big((\boldsymbol{C}_t'\boldsymbol{C}_t)^{-1}\boldsymbol{C}_t'(\boldsymbol{Q}_Z(t) \odot \boldsymbol{P}_Z(t)\Big) \div \Big(((\boldsymbol{C}_t'\boldsymbol{C}_t)^{-1}\boldsymbol{C}_t'\boldsymbol{P}_Z(t)\Big) \tag{133}$$

where $\div$ denotes element-wise division,[44] and $\boldsymbol{A}_t$ stems from the decomposition $\boldsymbol{B}_t = \boldsymbol{C}_t\boldsymbol{A}_t$ as in (128)–(130) Moreover, in the case where $\boldsymbol{C}_t$ is invertible, equation (133) can be further simplified

[44] Let $\boldsymbol{A}, \boldsymbol{B}$ be two vectors of same length, then $\boldsymbol{A} \div \boldsymbol{B} \equiv \text{diag}(\boldsymbol{B})^{-1}\boldsymbol{A}$, where $\text{diag}(\cdot)$ is the operator that transform a vector into a diagonal matrix.

as:

$$\underbrace{\Big(\boldsymbol{A}_t\big(\boldsymbol{Q}_S(t)\odot\boldsymbol{P}_S\big)\Big)\div\Big(\boldsymbol{A}_t\boldsymbol{P}_S\Big)}_{\text{Identified Counterfactual Outcomes}}=\underbrace{\Big(\boldsymbol{C}_t^{-1}\big(\boldsymbol{Q}_Z(t)\odot\boldsymbol{P}_Z(t)\big)\Big)\div\Big(\big(\boldsymbol{C}_t^{-1}\boldsymbol{P}_Z(t)\Big)}_{\text{Identification Formulas}} \tag{134}$$

The right-hand side of (134) summarizes all identified counterfactual outcomes. The left-hand side of (134) generates identification formulas. Equations (135)–(137) exemplify the left-hand side of (134) for t_h.

$$\boldsymbol{A}_{t_h}\big(\boldsymbol{Q}_S(t_h)\odot\boldsymbol{P}_S\big)=\begin{bmatrix} E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{fc})\,P(\boldsymbol{S}=\boldsymbol{s}_{fc})+E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{pl})\,P(\boldsymbol{S}=\boldsymbol{s}_{pl}) \\ E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ph})\,P(\boldsymbol{S}=\boldsymbol{s}_{ph}) \\ E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ah})\,P(\boldsymbol{S}=\boldsymbol{s}_{ah}) \end{bmatrix} \tag{135}$$

$$\boldsymbol{A}_{t_h}\boldsymbol{P}_S=\begin{bmatrix} P(\boldsymbol{S}=\boldsymbol{s}_{fc})+P(\boldsymbol{S}=\boldsymbol{s}_{pl}) \\ P(\boldsymbol{S}=\boldsymbol{s}_{ph}) \\ P(\boldsymbol{S}=\boldsymbol{s}_{ah}) \end{bmatrix} \tag{136}$$

$$\therefore\Big(\boldsymbol{A}_{t_h}\big(\boldsymbol{Q}_S(t_h)\odot\boldsymbol{P}_S\big)\Big)\div\Big(\boldsymbol{A}_{t_h}\boldsymbol{P}_S\Big)=\begin{bmatrix} E(Y(t_h)|\boldsymbol{S}\in\boldsymbol{s}_{fc},\boldsymbol{s}_{pl}) \\ E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ph}) \\ E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ah}) \end{bmatrix} \tag{137}$$

The right-hand side of (134) for t_h employs the matrix $\boldsymbol{C}_{t_h}^{-1}$ displayed in equation (138):

$$\boldsymbol{C}_{t_h}=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}\Rightarrow\boldsymbol{C}_{t_h}^{-1}=\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \tag{138}$$

Equations (139) and (140) exemplify the numerator and the denominators of the right-hand side of (134) for t_h :

$$\begin{aligned}\boldsymbol{C}_{t_h}^{-1}\boldsymbol{Q}_Z(t_h)\odot\boldsymbol{P}_Z(t_h)&=\begin{bmatrix} E(Y|T=t_h,Z=z_c)\,P(T=t_h|Z=z_c)-E(Y|T=t_h,Z=z_e)\,P(T=t_h|Z=z_e) \\ E(Y|T=t_h,Z=z_e)\,P(T=t_h|Z=z_e)-E(Y|T=t_h,Z=z_8)\,P(T=t_h|Z=z_8) \\ E(Y|T=t_h,Z=z_8)\,P(T=t_h|Z=z_8) \end{bmatrix},\\ &=\begin{bmatrix} E(Y\cdot D_{t_h}|Z=z_c)-E(Y\cdot\mathbf{1}[T=t_h]|Z=z_e) \\ E(Y\cdot D_{t_h}|Z=z_e)-E(Y\cdot\mathbf{1}[T=t_h]|Z=z_8) \\ E(Y\cdot D_{t_h}|Z=z_8) \end{bmatrix},\end{aligned} \tag{139}$$

$$\text{and }\boldsymbol{C}_{t_h}^{-1}\boldsymbol{P}_Z(t_h)=\begin{bmatrix} P(T=t_h|Z=z_c)-P(T=t_h|Z=z_e) \\ P(T=t_h|Z=z_e)-P(T=t_h|Z=z_8) \\ P(T=t_h|Z=z_8) \end{bmatrix}, \tag{140}$$

The final equation for t_h is presented in (141). The left-hand side of (141) lists all the identified counterfactual outcome means of $Y(t_h)$. The right-hand side provides the identification formulas that can be evaluated from observed data.

$$\therefore\underbrace{\begin{bmatrix} E(Y(t_h)|\boldsymbol{S}\in\boldsymbol{s}_{fc},\boldsymbol{s}_{pl}) \\ E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ph}) \\ E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{ah}) \end{bmatrix}}_{\boldsymbol{A}_{t_h}(\boldsymbol{Q}_S(t_h)\odot\boldsymbol{P}_S)\div\boldsymbol{A}_{t_h}\boldsymbol{P}_S}=\underbrace{\begin{bmatrix} \frac{E(Y\cdot D_{t_h}|Z=z_c)-E(Y\cdot D_{t_h}|Z=z_e)}{P(T=t_h|Z=z_c)-P(T=t_h|Z=z_e)} \\ \frac{E(Y\cdot D_{t_h}|Z=z_e)-E(Y\cdot D_{t_h}|Z=z_8)}{P(T=t_h|Z=z_e)-P(T=t_h|Z=z_8)} \\ \frac{E(Y\cdot D_{t_h}|Z=z_8)}{P(T=t_h|Z=z_8)} \end{bmatrix}}_{(\boldsymbol{C}_{t_h}^{-1}\boldsymbol{Q}_Z(t_h)\odot\boldsymbol{P}_Z(t_h))\div\boldsymbol{C}_{t_h}^{-1}\boldsymbol{P}_Z(t_h)}. \tag{141}$$

Equations (142)–(143) arise from applying formula (134) to t_m and t_l :

$$\underbrace{\begin{bmatrix} E(Y(t_m)|\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}\}) \\ E(Y(t_m)|\boldsymbol{S} = \boldsymbol{s}_{pm}) \\ E(Y(t_m)|\boldsymbol{S} = \boldsymbol{s}_{am}) \end{bmatrix}}_{\boldsymbol{A}_{t_m}(\boldsymbol{Q}_S(t_m)\odot \boldsymbol{P}_S) \div \boldsymbol{A}_{t_m}\boldsymbol{P}_S} = \underbrace{\begin{bmatrix} \frac{E(Y\cdot D_{t_m}|Z=z_8)-E(Y\cdot D_{t_m}|Z=z_c)}{P(T=t_m|Z=z_8)-P(T=t_m|Z=z_c)} \\ \frac{E(Y\cdot D_{t_m}|Z=z_c)-E(Y\cdot D_{t_m}|Z=z_e)}{P(T=t_m|Z=z_c)-P(T=t_m|Z=z_e)} \\ \frac{E(Y\cdot D_{t_m}|Z=z_e)}{P(T=t_m|Z=z_e)} \end{bmatrix}}_{\boldsymbol{C}_{t_m}^{-1}(\boldsymbol{Q}_Z(t_m)\odot \boldsymbol{P}_Z(t_m)) \div \boldsymbol{C}_{t_m}^{-1}\boldsymbol{P}_Z(t_m)}, \tag{142}$$

$$\underbrace{\begin{bmatrix} E(Y(t_l)|\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pm}\}) \\ E(Y(t_l)|\boldsymbol{S} = \boldsymbol{s}_{pl}) \\ E(Y(t_l)|\boldsymbol{S} = \boldsymbol{s}_{al}) \end{bmatrix}}_{\boldsymbol{A}_{t_l}(\boldsymbol{Q}_S(t_l)\odot \boldsymbol{P}_S) \div \boldsymbol{A}_{t_l}\boldsymbol{P}_S} = \underbrace{\begin{bmatrix} \frac{E(Y\cdot D_{t_l}|Z=z_e)-E(Y\cdot D_{t_l}|Z=z_8)}{P(T=t_l|Z=z_e)-P(T=t_l|Z=z_8)} \\ \frac{E(Y\cdot D_{t_l}|Z=z_8)-E(Y\cdot D_{t_l}|Z=z_c)}{P(T=t_l|Z=z_8)-P(T=t_l|Z=z_c)} \\ \frac{E(Y\cdot D_{t_l}|Z=z_c)}{P(T=t_l|Z=z_c)} \end{bmatrix}}_{\boldsymbol{C}_{t_l}^{-1}(\boldsymbol{Q}_Z(t_l)\odot \boldsymbol{P}_Z(t_l)) \div \boldsymbol{C}_{t_l}^{-1}\boldsymbol{P}_Z(t_l)}. \tag{143}$$

Response-type probabilities can be identified by equations $\boldsymbol{A}_t\boldsymbol{P}_S = \boldsymbol{C}_{t_h}^{-1}\boldsymbol{P}_Z(t_h)$ for $t = t_h, t_l, t_m$:

$$\therefore \underbrace{\begin{bmatrix} P(\boldsymbol{S} \in \boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}) \\ P(\boldsymbol{S} = \boldsymbol{s}_{ph}) \\ P(\boldsymbol{S} = \boldsymbol{s}_{ah}) \end{bmatrix}}_{\boldsymbol{A}_{t_h}\boldsymbol{P}_S} = \underbrace{\begin{bmatrix} P(T=t_h|Z=z_c) - P(T=t_h|Z=z_e) \\ P(T=t_h|Z=z_e) - P(T=t_h|Z=z_8) \\ P(T=t_h|Z=z_8) \end{bmatrix}}_{\boldsymbol{C}_{t_h}^{-1}\boldsymbol{P}_Z(t_h)}. \tag{144}$$

$$\underbrace{\begin{bmatrix} P(\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}\}) \\ P(\boldsymbol{S} = \boldsymbol{s}_{pm}) \\ P(\boldsymbol{S} = \boldsymbol{s}_{am}) \end{bmatrix}}_{\boldsymbol{A}_{t_m}\boldsymbol{P}_S} = \underbrace{\begin{bmatrix} P(T=t_m|Z=z_8) - P(T=t_m|Z=z_c) \\ P(T=t_m|Z=z_c) - P(T=t_m|Z=z_e) \\ P(T=t_m|Z=z_e) \end{bmatrix}}_{\boldsymbol{C}_{t_m}^{-1}\boldsymbol{P}_Z(t_m)}, \tag{145}$$

$$\underbrace{\begin{bmatrix} P(\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pm}\}) \\ P(\boldsymbol{S} = \boldsymbol{s}_{pl}) \\ P(\boldsymbol{S} = \boldsymbol{s}_{al}) \end{bmatrix}}_{\boldsymbol{A}_{t_l}\boldsymbol{P}_S} = \underbrace{\begin{bmatrix} P(T=t_l|Z=z_e) - P(T=t_l|Z=z_8) \\ P(T=t_l|Z=z_8) - P(T=t_l|Z=z_c) \\ P(T=t_l|Z=z_c) \end{bmatrix}}_{\boldsymbol{C}_{t_l}^{-1}\boldsymbol{P}_Z(t_l)}. \tag{146}$$

G.3 Estimating Identified Counterfactual Outcomes

We can connect the triangular property of the response matrices in (35), (42), and (43), to the IV literature of binary choice models. Similar to Imbens and Angrist (1994), counterfactual outcomes can be estimated by Two-Stage Least Squares (2SLS). The identification of $E(Y(t_l)|\boldsymbol{S} = \boldsymbol{s}_{pl})$ in (39) depends on z_8 and z_c. According to equation (143), the counterfactual is identified by the following equations:

$$E(Y(t_l)|\boldsymbol{S} = \boldsymbol{s}_{pl}) = \frac{E(Y \cdot D_{t_l}|Z = z_8) - E(Y \cdot D_{t_l}|Z = z_c)}{P(T = t_l|Z = z_8) - P(T = t_l|Z = z_c)}.$$

The equation is closely related with the LATE equation of Imbens and Angrist (1994). It can be estimated by the 2SLS (147)–(148) that regresses the choice indicator D_{t_l} on two IV indicators, $\mathbf{1}[Z = z_8]$ and $\mathbf{1}[Z = z_c]$ without a constant term (first stage) and then regresses the interaction

YD_{t_l} on a constant and the fitted values $\hat{D}_{t_l}$ (second stage):

$$\text{First Stage:} \qquad D_{t_l} = \gamma_1 \mathbf{1}[Z = z_8] + \gamma_2 \mathbf{1}[Z = z_c] + \epsilon_D \tag{147}$$

$$\text{Second Stage:} \quad YD_{t_l} = \beta_0 + \beta_{IV}\hat{D}_{t_l} + \epsilon_Y, \tag{148}$$

γ_1, γ_2 are linear coefficients of the first stage, β_0 is the intercept of the second stage, and β_{IV} is the linear coefficient that estimates $E(Y(t_l)|\boldsymbol{S} = \boldsymbol{s}_{pl})$. We can estimate different counterfactual outcomes by varying the IV-indicators and neighborhood choices as listed in Table A.8.

Table A.8: Two-Stage Least Square Estimation for Identified Parameters

Data Transformations				Identified Parameters
Endogenous Variables Choice Indicator	Dependent Variable Outcome Interaction	Instrumental Variable IV Indicators		
$D_{t_h} \equiv \mathbf{1}[T = t_h]$	$D_{t_h} \cdot Y$	$\mathbf{1}[Z = z_c]$ $\mathbf{1}[Z = z_8]$	$\mathbf{1}[Z = z_e]$ $\mathbf{1}[Z = z_e]$	$E(Y(t_h)\|S \in \{s_4, s_5\})$ $E(Y(t_h)\|S = s_7)$
$D_{t_m} \equiv \mathbf{1}[T = t_m]$	$D_{t_m} \cdot Y$	$\mathbf{1}[Z = z_c]$ $\mathbf{1}[Z = z_c]$	$\mathbf{1}[Z = z_8]$ $\mathbf{1}[Z = z_e]$	$E(Y(t_m)\|S \in \{s_4, s_7\})$ $E(Y(t_m)\|S = s_6)$
$D_{t_l} \equiv \mathbf{1}[T = t_l]$	$D_{t_l} \cdot Y$	$\mathbf{1}[Z = z_c]$ $\mathbf{1}[Z = z_8]$	$\mathbf{1}[Z = z_8]$ $\mathbf{1}[Z = z_e]$	$E(Y(t_l)\|S = s_5)$ $E(Y(t_l)\|S \in \{s_4, s_6\})$

This table lists the counterfactual outcome means estimated by 2SLS procedures. The first stage estimates use two IV indicators (columns 3 and 4) that are multiplied by γ_1, γ_2 in (147). The choice indicator (column 1) is the endogenous variable estimated in the first stage (147). The second stage uses the interaction of the outcome and the choice indicator (column 2) as dependent variable and uses the estimate of the first stage, which is multiplied by the linear coefficient β_{IV} in . The last column lists the identified counterfactual outcome mean.

We can control for pre-program variables $\boldsymbol{X}$ parametrically by including them as covariates in the 2SLS regressions.[45] Abadie (2003) proposes a κ-weighting scheme that nonparametrically controls for baseline variables in the LATE model.[46] The triangular property in (35) and (42) enables us to extend Abadie's κ to the case of multiple choices.

Counterfactual outcome $E\left(Y(t_l)|\boldsymbol{S} = \boldsymbol{s}_{pl}\right)$ is identified as a ratio of two *matching estimators* that depend on z_8 and z_c. This counterfactual outcome can also be expressed in (149) as the expectation of the observed outcome Y multiplied by a weighting function $\kappa(t_l, \boldsymbol{s}_{pl})$ in (150) which depends on z_8 and z_c.[47]

$$E\left(Y(t_l)|\boldsymbol{S} = \boldsymbol{s}_{pl}\right) = E\left(Y \cdot \frac{\kappa(t_l, \boldsymbol{s}_{pl})}{E(\kappa(t_l, \boldsymbol{s}_{pl}))}\right), \tag{149}$$

$$\text{such that } \kappa(t_l, \boldsymbol{s}_{pl}) = D_{t_l}\left(\frac{\mathbf{1}[Z = z_8]}{P(Z = z_8|\boldsymbol{X})} - \frac{\mathbf{1}[Z = z_c]}{P(Z = z_c|\boldsymbol{X})}\right). \tag{150}$$

The κ-weighting in (150) can be evaluated from data; it consists of the choice indicator D_{t_l} multiplied by the difference between IV indicators of z_8 and z_c divided by their respective probabilities

[45] Angrist and Imbens (1995) show that the 2SLS estimate is a weighted average of the counterfactual outcomes conditioned on the covariates. Weights consist of the variance of the choice indicators conditioned on the covariates.

[46] Abadie (2003) shows that the counterfactual outcomes of the LATE model can be evaluated by a weighted average of the outcome across the whole population. He names the weighting functions κ.

[47] Equation (149) also holds if Y were to be replaced by any measurable function $g(Y, D_{t_l}, \boldsymbol{X})$. See Navjeevan, Pinto, and Santos (2020) for an extension of Abadie's (2003) kappa-weighting scheme for multiple choice models and for arbitrary monotonicity conditions.

conditional on baseline variables $\boldsymbol{X}$. $E\left(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{pl}\right)$ can be estimated by the sample counterpart of (149), that is, $\sum_i Y_i \cdot \omega_i$, where $\omega_i = \kappa_i(t_l, \boldsymbol{s}_{pl})/(\sum_i \kappa_i(t_l, \boldsymbol{s}_{pl}))$ are weights that sum to one and $\kappa_i(t_l, \boldsymbol{s}_{pl})$ is the κ-weight of family i.[48] Weights κ for counterfactual outcomes in Table A.8 can be obtained by replacing D_{t_l}, z_8, z_c in (150) by their corresponding neighborhood choice and IV indicators.

H Additional Analysis of the Partial Identification Problem

This section provides further information on the solution to the problem of partial identification in MTO. Section (H.1) estimates bounds for the partially identified effects. Section (H.2) presents the identification analyses for high and medium poverty neighborhoods in the same fashion as the results for low poverty neighborhood in Section 4.2. Section H.3 discusses the propensity score estimator in greater detail.

H.1 Estimating Bounds for Partially identified Effects

H.2 Identification Results for High and Medium Poverty Neighborhoods

The main paper describes the problem of partial identification for the low poverty neighborhood. We seek to decompose the identified counterfactual mean $E(Y(t_l)|\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pm}\})$ into $E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{fc})$ and $E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{pm})$. Table A.9 summarises the identification equations for this decomposition.

In the case of the choice of medium poverty neighborhood, t_m, we seek to decompose $E(Y(t_m)|\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{ph}\})$ into $E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{fc})$ and $E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{ph})$. Table A.10 summarises the identification equations for t_m.

In the case of the choice of medium poverty neighborhood, t_h, we seek to decompose $E(Y(t_h)|\boldsymbol{S} \in \{\boldsymbol{s}_{fc}, \boldsymbol{s}_{pl}\})$ into $E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{fc})$ and $E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{pl})$. Table A.11 summarises the identification equations for t_h.

[48] The practical use of the κ-weights is to evaluate causal parameters via conventional estimation procedures that reweighed data according to the estimated values of κ. An example of an estimation procedure for $E\left(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{pl}\right)$ is: (1) estimate $P(Z=z_8|\boldsymbol{X}), P(Z=z_c|\boldsymbol{X})$; (2) construct weights $\hat{\kappa}(t_l, \boldsymbol{s}_{pl})$ as in (150); (3) estimate β_1 in regression $Y \cdot D_{t_l} = \beta_0 + \beta_1 \hat{D}_{t_l} + \beta_2 \boldsymbol{X} + \epsilon_Y$ via weighted least squares (WLS) that employ weights $\hat{\kappa}(t_l, \boldsymbol{s}_{pl})$. The WLS solves the sample analog of $(\beta_0, \beta_1, \beta_2) = \arg\min_{b_0,b_1,b_2} E(\kappa \cdot g(Y, D, \boldsymbol{X}))$, where $g(Y, D, \boldsymbol{X}) = (YD_{t_l} - (b_0 + b_1 \hat{D}_{t_l} + b_2 \boldsymbol{X}))^2$.

Table A.9: Identification Formulas for $E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{fc})$ and $E(Y(t_l)|\boldsymbol{S}=\boldsymbol{s}_{pm})$

Counterfactual Outcome	Integral Representation	Function of Propensity Scores
$E(Y(t_l)\|\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pm}\})=$	$\frac{\int_{P_{t_l}(z_8)}^{P_{t_l}(z_e)} E(Y(t_l)\|U_{t_l}=u)du}{P_{t_l}(z_e)-P_{t_l}(z_8)}$	$\equiv g_{t_l}\Big(P_{t_l}(z_e),P_{t_l}(z_8)\Big)$
$E(Y(t_h)\|\boldsymbol{S}=\boldsymbol{s}_{fc})=$	$\frac{\int_{P_{t_h}(z_8)}^{p^*_{t_l}} E(Y(t_l)\|U_{t_l}=u)du}{p^*_{t_l}-P_{t_l}(z_8)}$	$\equiv g_{t_h}\Big(p^*_{t_l},P_{t_l}(z_8)\Big)$
$E(Y(t_h)\|\boldsymbol{S}=\boldsymbol{s}_{pm})=$	$\frac{\int_{p^*_{t_l}}^{P_{t_l}(z_e)} E(Y(t_h)\|U_{t_l}=u)du}{P_{t_h}(z_e)-p^*_{t_l}}$	$\equiv g_{t_l}\Big(P_{t_l}(z_e),p^*_{t_l}\Big)$
where $p^*_{t_l}$	$=P(\boldsymbol{S}\in\{\boldsymbol{s}_{al},\boldsymbol{s}_{pl},\boldsymbol{s}_{fc}\})\in\Big(P_{t_l}(z_8),P_{t_l}(z_e)\Big)$	
because $P_{t_l}(z_8)$	$=P(\boldsymbol{S}\in\{\boldsymbol{s}_{al},\boldsymbol{s}_{pl}\})$ and $P_{t_l}(z_e)=P(\boldsymbol{S}\in\{\boldsymbol{s}_{al},\boldsymbol{s}_{pl},\boldsymbol{s}_{fc},\boldsymbol{s}_{pm}\})$	

Table A.10: Identification Formulas for $E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{fc})$ and $E(Y(t_m)|\boldsymbol{S}=\boldsymbol{s}_{ph})$

Counterfactual Outcome	Integral Representation	Function of Propensity Scores
$E(Y(t_m)\|\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{ph}\})=$	$\frac{\int_{P_{t_m}(z_8)}^{P_{t_m}(z_c)} E(Y(t_m)\|U_{t_m}=u)du}{P_{t_m}(z_8)-P_{t_m}(z_c)}$	$\equiv g_{t_m}\Big(P_{t_m}(z_8),P_{t_m}(z_c)\Big)$
$E(Y(t_m)\|\boldsymbol{S}=\boldsymbol{s}_{fc})=$	$\frac{\int_{P_{t_m}(z_8)}^{p^*_{t_m}} E(Y(t_m)\|U_{t_m}=u)du}{p^*_{t_m}-P_{t_m}(z_c)}$	$\equiv g_{t_m}\Big(p^*_{t_m},P_{t_m}(z_8)\Big)$
$E(Y(t_m)\|\boldsymbol{S}=\boldsymbol{s}_{ph})=$	$\frac{\int_{p^*_{t_m}}^{P_{t_m}(z_8)} E(Y(t_m)\|U_{t_m}=u)du}{P_{t_m}(z_8)-p^*_{t_m}}$	$\equiv g_{t_m}\Big(P_{t_m}(z_8),p^*_{t_m}\Big)$
where $p^*_{t_m}$	$=P(\boldsymbol{S}\in\{\boldsymbol{s}_{am},\boldsymbol{s}_{pm},\boldsymbol{s}_{fc}\})\in\Big(P_{t_m}(z_c),P_{t_m}(z_8)\Big)$	
because $P_{t_m}(z_e)$	$=P(\boldsymbol{S}\in\{\boldsymbol{s}_{am},\boldsymbol{s}_{pm}\})$ and $P_{t_m}(z_c)=P(\boldsymbol{S}\in\{\boldsymbol{s}_{am},\boldsymbol{s}_{pm},\boldsymbol{s}_{fc},\boldsymbol{s}_{ph}\})$	

Table A.11: Identification Formulas for $E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{fc})$ and $E(Y(t_h)|\boldsymbol{S}=\boldsymbol{s}_{pl})$

Counterfactual Outcome	Integral Representation	Function of Propensity Scores
$E(Y(t_h)\|\boldsymbol{S}\in\{\boldsymbol{s}_{fc},\boldsymbol{s}_{pl}\})=$	$\frac{\int_{P_{t_h}(z_e)}^{P_{t_h}(z_c)} E(Y(t_h)\|U_{t_h}=u)du}{P_{t_h}(z_c)-P_{t_h}(z_e)}$	$\equiv g_{t_h}\Big(P_{t_h}(z_c),P_{t_h}(z_e)\Big)$
$E(Y(t_h)\|\boldsymbol{S}=\boldsymbol{s}_{fc})=$	$\frac{\int_{P_{t_h}(z_e)}^{p^*_{t_h}} E(Y(t_h)\|U_{t_h}=u)du}{p^*_{t_h}-P_{t_h}(z_e)}$	$\equiv g_{t_h}\Big(p^*_{t_h},P_{t_h}(z_e)\Big)$
$E(Y(t_h)\|\boldsymbol{S}=\boldsymbol{s}_{pl})=$	$\frac{\int_{p^*_{t_h}}^{P_{t_h}(z_c)} E(Y(t_h)\|U_{t_h}=u)du}{P_{t_h}(z_c)-p^*_{t_h}}$	$\equiv g_{t_h}\Big(P_{t_h}(z_c),p^*_{t_h}\Big)$
where $p^*_{t_h}$	$=P(\boldsymbol{S}\in\{\boldsymbol{s}_{ah},\boldsymbol{s}_{ph},\boldsymbol{s}_{fc}\})\in\Big(P_{t_h}(z_e),P_{t_h}(z_c)\Big)$	
because $P_{t_h}(z_e)$	$=P(\boldsymbol{S}\in\{\boldsymbol{s}_{ah},\boldsymbol{s}_{ph}\})$ and $P_{t_h}(z_c)=P(\boldsymbol{S}\in\{\boldsymbol{s}_{ah},\boldsymbol{s}_{ph},\boldsymbol{s}_{fc},\boldsymbol{s}_{pl}\})$	

H.3 Nonparametric Propensity Score Estimator with Covariates

As mentioned in the main paper, the response matrix of MTO implies the unordered monotonicity conditions, which in turn is equivalent to state that the choice indicator can be expressed by the following inequality $D_t = \mathbf{1}[P_t(Z) \geq U_t]$ such that $U_t \sim Unif[0,1]$ and for $t \in \{t_h, t_m, t_l\}$ (Heckman and Pinto, 2018).

The main paper shows that the counterfactual outcome $E(Y(t_l)|\boldsymbol{S} = \boldsymbol{s}_{pl})$ can be expressed as:

$$E(Y(t_l)|\boldsymbol{S} = \boldsymbol{s}_{pl}) = \frac{E(YD_{t_l}|Z = z_8) - E(YD_{t_l}|Z = z_c)}{E(D_{t_l}|Z = z_8) - E(D_{t_l}|Z = z_c)} = \frac{\int_{P_{t_l}(z_c)}^{P_{t_l}(z_8)} E(Y(t_l)|U_{t_l} = u)du}{P_{t_l}(z_8) - P_{t_l}(z_c)}, \tag{151}$$

where $P_t(z) \equiv P(T = t|Z = z)$ is the unconditional propensity score for $t \in \{t_h, t_m, t_l\}$ and $z \in \{z_c, z_8, z_e\}$.

Consider a more general terminology in which the values $z, z' \in \{z_c, z_8, z_e\}$ and the choice value $t \in \{t_h, t_m, t_l\}$ are such that such that $P_t(z') > P_t(z)$ are associated with a response-type $\boldsymbol{s} \in supp(\boldsymbol{S})$ for which (152) holds.

$$E(Y(t)|\boldsymbol{S} = \boldsymbol{s}) = \frac{\int_{P_t(z)}^{P_t(z')} E(Y(t)|U_t = u)du}{P_t(z) - P_t(z')}, \tag{152}$$

In summary, equation (151) simply describes a connection between instrumental values z, z', neighborhood choice t and their associated with response-type $\boldsymbol{s} \in supp(\boldsymbol{S})$. The numerator in (152) is identified by:

$$\begin{aligned}
&\int_{P_t(z)}^{P_t(z')} E(Y(t)|U_t = u)du = \\
&\quad = E(Y(t)\mathbf{1}[P_t(z) \leq U_t \leq P_t(z')]) \\
&\quad = E(Y(t)\mathbf{1}[U_t \leq P_t(z')] - \mathbf{1}[U_t \geq P_t(z)]) \\
&\quad = E(Y(t)\mathbf{1}[U_t \leq P_t(Z)]|P_t(Z) = P_t(z')) - E(Y(t)\mathbf{1}[U_t \geq P_t(Z)]|P_t(Z) = P_t(z)) \\
&\quad = E(YD_t|P_t(Z) = P_t(z')) - E(YD_t|P_t(Z) = P_t(z))
\end{aligned} \tag{153}$$

This section seeks to identify (152) as a function of propensity scores conditional of $\boldsymbol{X}$, that is $P_t(z, \boldsymbol{x}) \equiv P(T = t|Z = z, \boldsymbol{X} = \boldsymbol{x})$. The conditional version of (153) is given by:

$$\begin{aligned}
E(Y(t)|\boldsymbol{S} = \boldsymbol{s}, \boldsymbol{X} = \boldsymbol{x}) &= \frac{\int_{P_t(z',\boldsymbol{x})}^{P_t(z,\boldsymbol{x})} E(Y(t)|U_t = u, \boldsymbol{X} = \boldsymbol{x})du}{P_t(z, \boldsymbol{x}) - P_t(z', \boldsymbol{x})} \\
&= \frac{E(YD_t|P_t(Z) = P_t(z', \boldsymbol{x}), \boldsymbol{X} = \boldsymbol{x}) - E(YD_t|P_t(Z) = P_t(z, \boldsymbol{x}), \boldsymbol{X} = \boldsymbol{x})}{P_t(z', \boldsymbol{x}) - P_t(z, \boldsymbol{x})}
\end{aligned} \tag{154}$$

Integrating $E(Y(t)|\boldsymbol{S} = \boldsymbol{s}, \boldsymbol{X} = \boldsymbol{x})$ over $\boldsymbol{X}$ generates the following equation:

$$\begin{aligned}
\int E(Y(t)|\boldsymbol{S} = \boldsymbol{s}, \boldsymbol{X} = \boldsymbol{x})dF_{\boldsymbol{X}|\boldsymbol{S}=\boldsymbol{s}}(\boldsymbol{x}) &= \int E(Y(t)|\boldsymbol{S} = \boldsymbol{s}, \boldsymbol{X} = \boldsymbol{x})dF_{\boldsymbol{X}|\boldsymbol{S}=\boldsymbol{s}}(\boldsymbol{x}) \\
&= \int E(Y(t)|\boldsymbol{S} = \boldsymbol{s}, \boldsymbol{X} = \boldsymbol{x})\frac{P(\boldsymbol{S} = \boldsymbol{s}|\boldsymbol{X} = \boldsymbol{x})}{P(\boldsymbol{S} = \boldsymbol{s})}dF_{\boldsymbol{X}}(\boldsymbol{x}),
\end{aligned} \tag{155}$$

where the second equality is due to Bayes' theorem. Recall that $P(\boldsymbol{S} = \boldsymbol{s}|\boldsymbol{X} = \boldsymbol{x}) = P_t(z', \boldsymbol{x}) - P_t(z, \boldsymbol{x})$ and thereby $P(\boldsymbol{S} = \boldsymbol{s}) = \int P_t(z', \boldsymbol{x}) - P_t(z, \boldsymbol{x})dF_{\boldsymbol{X}}(\boldsymbol{x})$. Inserting (154) into (155) and using the above results generates:

$$E(Y(t)|\boldsymbol{S}=\boldsymbol{s}) = \\ = \frac{\int \Big(E(YD_t|P_t(Z)=P_t(z',\boldsymbol{x}),\boldsymbol{X}=\boldsymbol{x}) - E(YD_t|P_t(Z)=P_t(z,\boldsymbol{x}),\boldsymbol{X}=\boldsymbol{x}) \Big) dF_{\boldsymbol{X}}(\boldsymbol{x})}{\int P_t(z',\boldsymbol{x}) - P_t(z,\boldsymbol{x}) dF_{\boldsymbol{X}}(\boldsymbol{x})} \tag{156}$$

I Sensitivity Analyses

This section present additional evaluations that check the robustness of these findings under modifications of the baseline model. I am presenting only the estimations regarding the causal effects for full compliers $\boldsymbol{s}_{fc}$ in order to satisfy the maximum limit of 25 pages for this Appendix. The estimates for counterfactual outcomes, TOT decomposition, additional empirical analyses and specification tests can be obtained by request (rodrig@econ.ucla.edu).

Tables A.12–A.14 presents results based on variations of the original model that generates Table 7 of the main text.

Table A.12 suppresses the interaction of site fixed effects and the propensity scores. Suppressing the interaction between propensity scores and site fixed effects forces cities to shift the the mean potential outcomes for a given neighborhood choice in parallel across all response-types.

Table A.13 suppresses the interaction of baseline variables and propensity scores. This forces that the family baseline characteristics to shift the mean potential outcomes for a given neighborhood choice in parallel.

Table A.14 estimates the same outcome equation displayed in the main text. The model however uses a multinomial logit model to estimate propensity scores, instead of the linear probability model in (58).

Estimates in Tables A.12–A.14 are very close to those presented in Table 7.

Table A.12: Causal Effects for Full Compliers $\boldsymbol{S} = \boldsymbol{s}_4$ (No Site Interaction)

	$E(Y(t_l) - Y(t_h)\|\boldsymbol{s}_4)$		$E(Y(t_l) - Y(t_m)\|\boldsymbol{s}_4)$		$E(Y(t_m) - Y(t_h)\|\boldsymbol{s}_4)$	
Income of Family Head	2.030	***	0.110		1.919	**
(s.e.)	0.761		0.896		0.902	
(p-value)	0.005		0.905		0.032	
Income of Head and Spouse	0.710		0.486		0.224	
(s.e.)	0.826		0.899		1.019	
(p-value)	0.393		0.598		0.800	
Total household income	1.421		1.083		0.337	
(s.e.)	0.871		0.981		1.060	
(p-value)	0.117		0.277		0.752	
Above Poverty Line	0.089	**	0.038		0.051	
(s.e.)	0.039		0.050		0.053	
(p-value)	0.018		0.453		0.348	
Employed without welfare	0.102	**	0.034		0.068	
(s.e.)	0.044		0.057		0.059	
(p-value)	0.027		0.570		0.245	
Currently on welfare	−0.111	***	0.061		−0.172	***
(s.e.)	0.041		0.055		0.057	
(p-value)	0.008		0.270		0.010	
Job tenure	0.073		0.028		0.044	
(s.e.)	0.044		0.052		0.053	
(p-value)	0.102		0.607		0.393	
Economic self-sufficiency	0.062	*	−0.023		0.085	*
(s.e.)	0.033		0.045		0.045	
(p-value)	0.062		0.592		0.055	
Neighborhood Poverty	−32.843	***	−20.999	***	−11.844	***
(s.e.)	0.996		1.690		1.914	
(p-value)	0.000		0.000		0.000	

This table evaluates the neighborhood effects for full compliers $\boldsymbol{s}_4$ across several outcomes. The first column lists the outcome variables. The second column evaluates the causal effect between the neighborhood types of low and high poverty. The third column compares low versus medium poverty neighborhoods and the last column evaluates the neighborhood effects between medium versus high poverty types. The results are based on a semi-parametric method that evaluates propensity scores and response-type probabilities using a linear probability model. All estimates are conditioned on the site of intervention and account for the person-level weight for adult survey of the interim analyses (Interim Impacts Evaluation manual, 2003, Appendix B). Inference is obtained by a bootstrap method that employs a weighted sampling scheme. The p-values are associated with the double-tailed inference that tests if the estimates are equal to zero. Asterisks indicate the typical p-value thresholds: *** for p-value < 0.01, ** for $0.01 \leq p$-value < 0.05, * for $0.05 \leq p$-value < 0.1.

Table A.13: Causal Effects for Full Compliers $\boldsymbol{S} = \boldsymbol{s}_4$ (No Covariate Interaction)

	$E(Y(t_l) - Y(t_h)\|\boldsymbol{s}_4)$		$E(Y(t_l) - Y(t_m)\|\boldsymbol{s}_4)$		$E(Y(t_m) - Y(t_h)\|\boldsymbol{s}_4)$	
Income of Family Head	2.188	***	−0.289		2.477	
(s.e.)	0.822		1.516		1.469	
(p-value)	0.005		0.847		0.112	
Income of Head and Spouse	0.865		0.176		0.689	
(s.e.)	0.862		1.642		1.711	
(p-value)	0.335		0.910		0.663	
Total household income	2.040	**	0.693		1.347	
(s.e.)	0.908		1.621		1.623	
(p-value)	0.035		0.647		0.420	
Above Poverty Line	0.122	***	−0.050		0.173	*
(s.e.)	0.042		0.086		0.084	
(p-value)	0.008		0.607		0.082	
Employed without welfare	0.114	**	0.140		−0.026	
(s.e.)	0.046		0.095		0.097	
(p-value)	0.022		0.157		0.803	
Currently on welfare	−0.130	***	−0.048		−0.081	
(s.e.)	0.044		0.090		0.089	
(p-value)	0.007		0.565		0.343	
Job tenure	0.094	**	0.061		0.033	
(s.e.)	0.047		0.096		0.094	
(p-value)	0.048		0.533		0.723	
Economic self-sufficiency	0.077	**	−0.133		0.210	**
(s.e.)	0.034		0.087		0.084	
(p-value)	0.027		0.177		0.030	
Neighborhood Poverty	−33.283	***	−21.631	***	−11.652	***
(s.e.)	1.008		2.216		2.279	
(p-value)	0.000		0.000		0.000	

This table evaluates the neighborhood effects for full compliers $\boldsymbol{s}_4$ across several outcomes. The first column lists the outcome variables. The second column evaluates the causal effect between the neighborhood types of low and high poverty. The third column compares low versus medium poverty neighborhoods and the last column evaluates the neighborhood effects between medium versus high poverty types. The results are based on a semi-parametric method that evaluates propensity scores and response-type probabilities using a linear probability model. All estimates are conditioned on the site of intervention and account for the person-level weight for adult survey of the interim analyses (Interim Impacts Evaluation manual, 2003, Appendix B). Inference is obtained by a bootstrap method that employs a weighted sampling scheme. The p-values are associated with the double-tailed inference that tests if the estimates are equal to zero. Asterisks indicate the typical p-value thresholds: *** for p-value < 0.01, ** for $0.01 \leq p$-value < 0.05, * for $0.05 \leq p$-value < 0.1.

Table A.14: Causal Effects for Full Compliers $\boldsymbol{S} = \boldsymbol{s}_4$ (Using Multinomial Logit)

	$E(Y(t_l) - Y(t_h)\|\boldsymbol{s}_4)$		$E(Y(t_l) - Y(t_m)\|\boldsymbol{s}_4)$		$E(Y(t_m) - Y(t_h)\|\boldsymbol{s}_4)$	
Income of Family Head	2.471	***	0.887		1.585	
(s.e.)	0.821		1.179		1.225	
(p-value)	0.005		0.453		0.220	
Income of Head and Spouse	1.298		1.278		0.021	
(s.e.)	0.923		1.371		1.399	
(p-value)	0.195		0.350		0.993	
Total household income	2.161	**	2.780	**	−0.619	
(s.e.)	0.954		1.379		1.423	
(p-value)	0.032		0.048		0.668	
Above Poverty Line	0.133	***	0.077		0.057	
(s.e.)	0.048		0.069		0.065	
(p-value)	0.010		0.275		0.408	
Employed without welfare	0.117	**	0.096		0.021	
(s.e.)	0.051		0.079		0.079	
(p-value)	0.023		0.223		0.808	
Currently on welfare	−0.108	**	−0.051		−0.057	
(s.e.)	0.046		0.069		0.068	
(p-value)	0.028		0.462		0.372	
Job tenure	0.120	**	0.053		0.067	
(s.e.)	0.050		0.077		0.079	
(p-value)	0.028		0.457		0.398	
Economic self-sufficiency	0.076	*	−0.044		0.119	*
(s.e.)	0.042		0.059		0.057	
(p-value)	0.087		0.485		0.067	
Neighborhood Poverty	−32.893	***	−22.737	***	−10.157	***
(s.e.)	1.150		1.725		1.995	
(p-value)	0.000		0.000		0.000	

This table evaluates the neighborhood effects for full compliers $\boldsymbol{s}_4$ across several outcomes. The first column lists the outcome variables. The second column evaluates the causal effect between the neighborhood types of low and high poverty. The third column compares low versus medium poverty neighborhoods and the last column evaluates the neighborhood effects between medium versus high poverty types. The results are based on a semi-parametric method that evaluates propensity scores and response-type probabilities using a multinomial logit model. All estimates are conditioned on the site of intervention and account for the person-level weight for adult survey of the interim analyses (Interim Impacts Evaluation manual, 2003, Appendix B). Inference is obtained by a bootstrap method that employs a weighted sampling scheme. The p-values are associated with the double-tailed inference that tests if the estimates are equal to zero. Asterisks indicate the typical p-value thresholds: *** for $p\text{-value} < 0.01$, ** for $0.01 \leq p\text{-value} < 0.05$, * for $0.05 \leq p\text{-value} < 0.1$.